BBSM Neutrino Course

Lecture XXV

LMU

Spring 2020

LR symmetry : Consequences

$\underline{P}$:

$$\boxed{f_L \longleftrightarrow f_R}$$

$$f_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix} \Updownarrow \quad f_R = \begin{pmatrix} 0 \\ u_R \end{pmatrix}$$

$$u_L \longleftrightarrow u_R$$

C:

$$f_L \longleftrightarrow \text{cont. } f_R^*$$

$$f_L \longleftrightarrow C\,\overline{f_R}^T = (f^c)_L$$

$$\underbrace{\boxed{L \longleftrightarrow L}}$$

$$\boxed{\text{C can be gauged}}$$

D: $\phi \longrightarrow -\phi \quad (\phi \in \mathbb{R})$

$$\Downarrow$$

domain walls

<u>SU(2):</u> $\Phi \rightarrow U\,\Phi\,, \quad U^{+}U = 1$

$$U = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \in SU(2)$$

$$= e^{i\pi\sigma_3} = e^{\underbrace{2i\pi\, T_3}}$$

$$\Downarrow$$

$$\boxed{\underline{\text{no}}\ \text{domain walls}}$$

(a) $\mathcal{M}_\infty \longrightarrow \mathcal{M}_0 = \{v, -v\}$

$\mathbb{Z}_2 \equiv D$

$$\mathcal{M}_\infty \longrightarrow \mathcal{M}_0(SU(2)) = S_3$$

(b) $\phi_0 = h + iG$

$\boxed{\text{gauge } G}$

$$SU(2)_L \times SU(2)_R \times \underbrace{U(1)_{B-L} \times SU(3)_c}$$

$$\subseteq SU(2)_L \times SU(2)_R \times \underbrace{SU(4)_c}_{\text{Pati-Salam}}$$

$$\begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_{L,R}$$

$\to$ violet

$$SU(4)_c \xrightarrow[\substack{\text{PS scale} \\ (M_{PS})}]{} SU(3)_c \times U(1)_{B-L}$$

$$SO(4) \times SU(4) = SO(6)$$

$\parallel$

$SU(2) \times SU(2)$

$r = 2$, # of gen $= 6$

$r = 3$

# of gen $= 15$

$4^2 - 1 = \frac{6 \cdot 5}{2}$

$\Downarrow$

$$LR \subseteq \underbrace{SO(6) \times SO(4)}_{\boxed{SO(10)}}$$

$\Downarrow$

fermions $=$ 16_F spinorial repr.

Lorentz $\Rightarrow$ (Euclidean) $SO(4)$

$$\{\gamma_i, \gamma_j\} = 2\delta_{ij}$$

$$\{\gamma_5, \gamma_i\} = 0, \quad \gamma_5^2 = 1$$

$\propto \gamma_1 \cdots \gamma_4$ $\qquad L(R) \equiv \frac{1 \pm \gamma_5}{2}$

$$\boxed{4 \text{ spinor} = 2_L + 2_R}$$

$$\Downarrow$$

$SO(10) \qquad a, b = 1, \ldots, 10$

$$\{\Gamma_a, \Gamma_b\} = 2\delta_{ab}$$

$$\Gamma_{FIVE} \equiv \Gamma_{11} = \cdot\cdot\, \Gamma_1 \cdots \Gamma_{10}$$

$$\Gamma_{FIVE}^2 = 1; \quad \{\Gamma_{FIVE}, \Gamma_a\} = 0$$

$$\Sigma_{ab} = \frac{1}{4i}[\Gamma_a, \Gamma_b]$$

$$[\Sigma_{ab}, \Gamma_{FIVE}] = 0$$

$$\boxed{2^5 = 32 = 16_+ + 16_-}$$

SM

$$\left.\begin{array}{cc} u_L^\alpha & u_R^\alpha \\ d_L^\alpha & d_R^\alpha \end{array}\right\} \begin{array}{c} 12 \\ =6+6 \end{array}$$

$$\begin{array}{ccc} e_L & e_R & 2 \\ \nu_L & \bullet & 1 \\ \hline & & 15 \end{array}$$

$$\boxed{16_F = 15 + 1}$$

$$\Downarrow$$

$$\boxed{\text{SM family} + \nu_R}$$

$$\Downarrow$$

$$16_F = \begin{pmatrix} u \\ u^c \\ d \\ d^c \\ e \\ e^c \\ \nu \\ \nu^c \end{pmatrix}_L$$

$$u_L^c = C\bar{u}_R^T$$

Lorentz

$$\boxed{\text{Symmetry} = SO(10) \times \text{Lorentz}}$$

<u>SU(2)</u> $\begin{pmatrix} u \\ d \end{pmatrix}$

$$\boxed{SO(10): \quad f \longleftrightarrow f^c \quad \text{gauged}}$$

$$\boxed{SU(2)_L \times SU(2)_R \times U(1)}$$

$$P: \quad f_L \longleftrightarrow f_R$$

$$\mathcal{L}_Y = \overline{f_L}\, \Phi\, Y f_R + \overline{f_R}\, \Phi^\dagger Y^\dagger f_L \qquad (1)$$

<u>SO(10)</u> $\boxed{\text{what about generation?}}$

$C = $ gauge $\quad \boxed{f_R \Leftrightarrow \overline{V}_R f_R}$

$$\left.\begin{aligned} f_{L,R} &\to U_{L,R}\, f_{L,R} \\ \Phi &\to U_L\, \Phi\, U_R^\dagger \end{aligned}\right\} \Rightarrow \boxed{\Phi \xrightarrow[P]{} \Phi^\dagger}$$

$$\overline{f_L}\, y\, \Phi\, f_R \xrightarrow{P} \overline{f_R}\, y\, \Phi^+ f_L \quad (2)$$

$$\boxed{(1)+(2) \Rightarrow y = y^+}$$

$$C: \quad f_L \hookrightarrow C\overline{f_R}^T \propto f_R^*$$

$$\Rightarrow \boxed{y = (y^+)^* = y^T}$$

$$\Phi \longrightarrow (\Phi^+)^* = \Phi^T$$

$$\mathcal{L}_y = \overline{f_L}\, y\, \Phi\, f_R \xrightarrow{C}$$

$$\longrightarrow \overline{C\overline{f_R}^T}\, y\, \Phi'\, C\overline{f_L}^T \quad (3)$$

$$\Downarrow$$

$$(3) = \left[C\left(f_R^{+}\gamma^0\right)^T\right]^{+}\gamma^0\, y\, \Phi'\, C\, \bar{f}_L^{\,T}$$

$$= \left(C\,\gamma_0\, f_R^{*}\right)^{+}\gamma^0\, y\, \bar{\Phi}'\, C\, \bar{f}_L^{\,T}$$

$$= f_R^T\, \gamma_0\, C^{+}\gamma^0\, y\, \bar{\Phi}'\, C\, \bar{f}_L^{\,T}$$

$$= f_R^T\, \underset{\underset{(-1)}{\shortparallel}}{C^2}\, y\, \bar{\Phi}'\, \bar{f}_L^{\,T} =$$

$$= \underbrace{\bar{f}_L\, y^T\, \bar{\Phi}'^{\,T} f_R} \qquad (4)$$

$$\bar{f}_L\, y\, \bar{\Phi}\, f_R \Rightarrow \quad \phi'^T = \phi$$

$$\Downarrow$$

$$\boxed{\phi' = \phi^T,\; y = y^T}$$

$$\boxed{\mathcal{P}: \quad Y = \mathcal{Y}^{+}, \quad \Phi \leftrightarrow \Phi^{+}}$$

$$C: \quad Y = \mathcal{Y}^{T}, \quad \Phi \leftrightarrow \Phi^{+}$$

Physics

SM $\Leftrightarrow$ weak int.

P = broken maximally

$\Rightarrow C$ = — " —

$G_{LR} \times \mathcal{P}\ (C)$

$\mathcal{P} \sim C$

$$\mathcal{L}_Y(\Delta) = \boxed{\ell_L^T C\, i\sigma_2\, \Delta_L Y_{\Delta_L}\, \ell_L}$$

$$+\ \ell_R^T C\, i\sigma_2\, \Delta_R\, Y_{\Delta_R}\, \ell_R\ \text{h.c.}$$

$\mathcal{P}$: $\quad \ell_L \longleftrightarrow f_R$

$\quad \Longrightarrow \Delta_L \longrightarrow \Delta_R \Rightarrow Y_{\Delta_L} = Y_{\Delta_R}$

C: $\quad \ell_L \longleftrightarrow \alpha f_R^*$

$\quad \Delta_L \longleftarrow \Delta_R^* \Longrightarrow Y_{\Delta_L} = Y_{\Delta_R}^*$

$\downarrow$

Neutrino

$$\underline{M}_\nu = - \underline{M}_D^T \frac{1}{\underline{M}_N} \underline{M}_D$$

$$\underline{M}_N \equiv \underline{M}_{\nu_R}^*$$

$$\left(N = C \bar{\nu}_R^T \propto \nu_R^* \right)$$

$$\underline{M}_{\nu_R} = y_{\Delta_R} v_R$$

$$v_R = \langle \Delta_R \rangle$$

$$\langle \Delta_L \rangle = 0 + O\left(\frac{1}{v_R} \right)$$

prior to SM breaking

$$\boxed{M_{\nu_L}^{(direct)} = y_{\Delta_L} v_L}$$

$$\Downarrow$$

$$C: \quad M_{\nu_L}^{(direct)} = y_{\Delta_R}^* v_L$$

$$= \left(y_{\Delta_R}^* v_R\right) \frac{v_L}{v_R}$$

$$= M_{\nu_R}^* \left(\frac{v_L}{v_R}\right)$$

$$= M_N \frac{v_L}{v_R}$$

$$\Downarrow \quad \boxed{\text{prediction}}$$

$$\boxed{M_\nu = M_N \frac{v_L}{v_R} - M_D^T \frac{1}{M_N} M_D} \quad (*)$$

$\Downarrow$ task $\boxed{M_D \propto v_R}$

compute M_D !!!

$$\boxed{\theta_{\nu N} = \frac{1}{M_N} M_D} \quad \boxed{\text{mixing}}$$

$$v_R \to \infty \Rightarrow M_\nu \to 0$$

$$(M_N \to \infty,\ v_L \to 0)$$

$$\boxed{\text{smallness of } m_\nu \Leftrightarrow \text{(near) maximality } \not{P}}$$

$\Updownarrow$

$$M_{\nu N} = \begin{array}{c} \nu \\ N \end{array} \begin{pmatrix} M_N \frac{v_L}{v_R} & M_D^T \\ M_D & M_N \end{pmatrix}$$

$\Downarrow$ (3) divide by M_N

$$\boxed{\frac{1}{M_N} M_\nu = \frac{v_L}{v_R} - \frac{1}{M_N} M_D^T \frac{1}{M_N} M_D}$$

**

$$M_D = y_D^T v$$

$$y^T = y \qquad (c)$$

$\Downarrow$

$$\left[M_D^T = M_D \right]$$

$\Downarrow$

$$\boxed{\frac{1}{M_N} M_\nu - \frac{\nu_L}{\nu_R} = -\underbrace{\frac{1}{M_N} M_D \frac{1}{M_N} M_D}_{***}}$$

$\Downarrow$

$$\boxed{\left(\frac{1}{M_N} M_D\right)^2 = -\left(\frac{1}{M_N} M_\nu - \frac{\nu_2}{\nu_R}\right)}$$

$\Downarrow$

$$\boxed{M_D = i M_N \sqrt{\frac{1}{M_N} M_\nu - \frac{\nu_2}{\nu_R}}}$$

$\Downarrow$

$$\boxed{\Theta = i \sqrt{\frac{1}{M_N} M_\nu - \frac{\nu_2}{\nu_R}}}$$

$$\boxed{\text{to "solve" seesaw} \iff \text{to know } M_\nu,\ M_N}$$

<u>seesaw</u>: $M_\nu = -M_D^T \dfrac{1}{M_N} M_D$

$$\Rightarrow\ M_D = i\sqrt{M_N}\ O\ \sqrt{M_\nu}$$

$$O^T O = 1$$

arbitrary

$$\Downarrow$$

$$\boxed{O_{(LR)} = \text{fixed}}$$

$\boxed{\text{quark sector}}$

$$M_u = U_{Lu}^{\dagger}\, m_u\, V_{Ru}$$

$$M_D = V_{Ld}^{\dagger}\, m_d\, U_{Rd}$$

m_u, m_d = diagonal matrices

$$\boxed{V_{CKM} = U_{Lu}^{\dagger}\, U_{Ld}}$$

$\boxed{\text{lepton sector}}$

$$\boxed{\begin{array}{l} u \leftrightarrow \nu \\ d \leftrightarrow e \end{array}}$$

$\boxed{\text{basis = charged leptons diagonal}}$

$$V_q \equiv V_{CKM} = V_{PMNS} \equiv V_\ell$$

$$\boxed{M_\nu = V_{L\ell}{}^T m_\nu \, V_{L\ell}}$$

$\parallel$ symmetric (Majorana) $\quad$ diagonal

$$V_{L\ell} \equiv V_L \quad (\text{leptonic})$$

$$V_{R\ell} \equiv V_R \quad (-\text{''}-)$$

$$\boxed{M_N = V_R{}^T m_N V_R}$$

$$\boxed{\begin{aligned} \underline{M}_\nu &= V_L^T m_\nu V_L \\ M_N &= V_R^T m_N V_R \end{aligned}}$$

$$\underline{M}_\nu \Longleftrightarrow m_\nu, V_L$$

neutrino oscillations

<u>Soln</u> : $\Delta m_\odot^2 \simeq 10^{-5}\, eV^2$

$\nu_e \to \nu_\mu$ $\boxed{\theta_\odot \simeq 30^\circ \ (\theta_{12})}$

<u>ATM</u> : $\sim \Delta m^2_{ATM} \simeq 10^{-3}\, eV^2$

$\nu_\mu \to \nu_\tau$ $\theta_{ATM} \simeq 45^\circ \ (\theta_{23})$

$$\boxed{\theta_{13} \simeq 10^\circ}$$

$$\downarrow$$

$$\boxed{m_\nu \gtrsim 1/25 \ \mathrm{eV}}$$

$$\boxed{m_\nu = ?}$$

KATRIN

(β decay)

$\updownarrow$

direct

$\downarrow$

probes $\overset{=\text{Dirac}}{m_D}$, m_M = Majorana

$$\boxed{0\nu 2\beta}$$

m_M = Majorana

$$\boxed{M_N = V_R^T m_N V_R \quad ???}$$

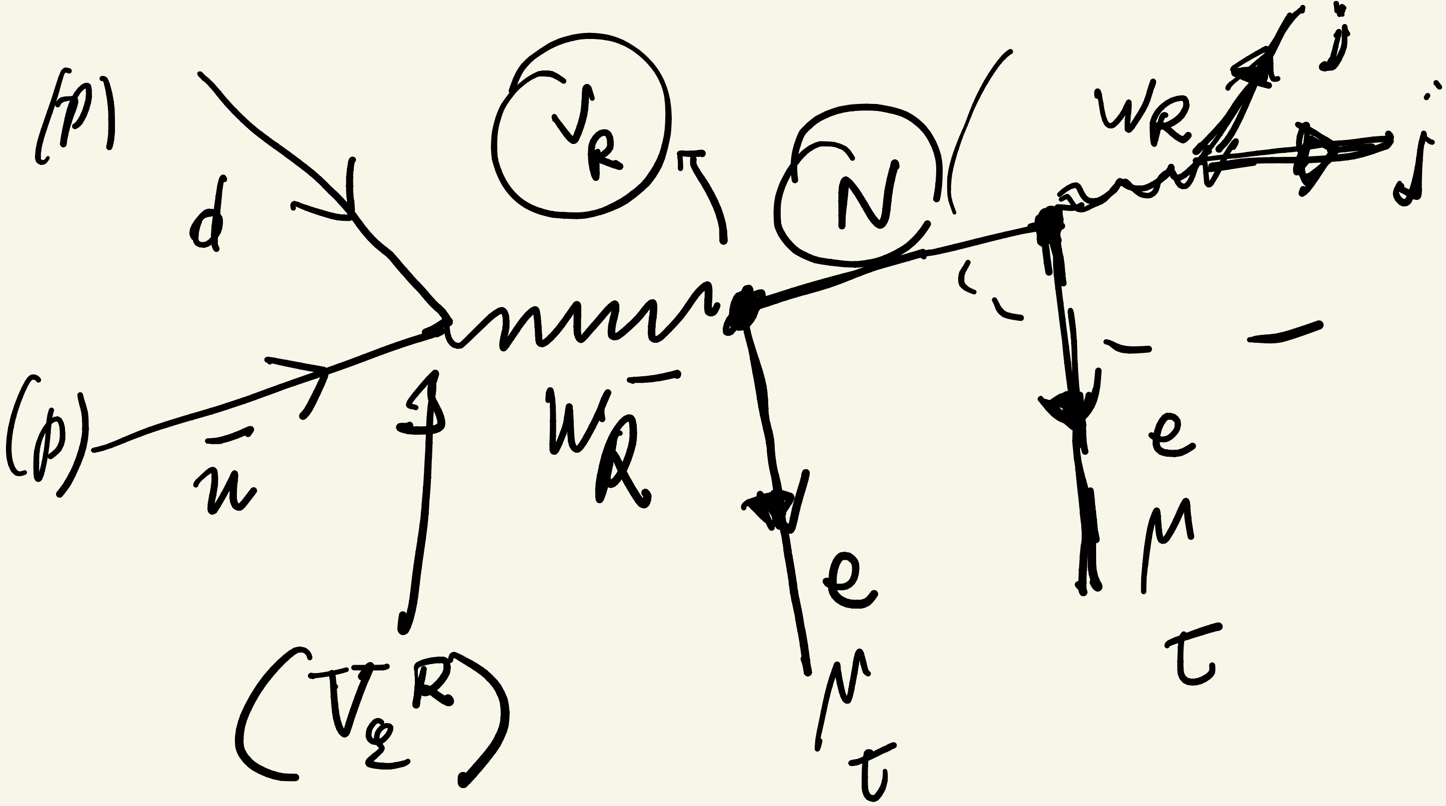
$$\boxed{m_N > M_W = M_{W_L}}$$

$$\boxed{\Theta_{\nu N} \; \bar{N} e W \leftarrow \theta^2 \simeq m_\nu / m_N}$$

$\boxed{\text{small effect}}$

$N = \nu_R^* \Longleftrightarrow W_R$ to produce

$$\Downarrow$$



m_{W_R} (m_N) = measure E, P of final states

$$\boxed{\text{measure } V_R,\ m_N}$$

N = Majorana?

$$N \to e_R + (q + q')$$

$$N \to \bar{e}_R + (q'' + q''')$$

$$\boxed{\Gamma(N \to e) = \Gamma(N \to \bar{e})}$$

Majorana

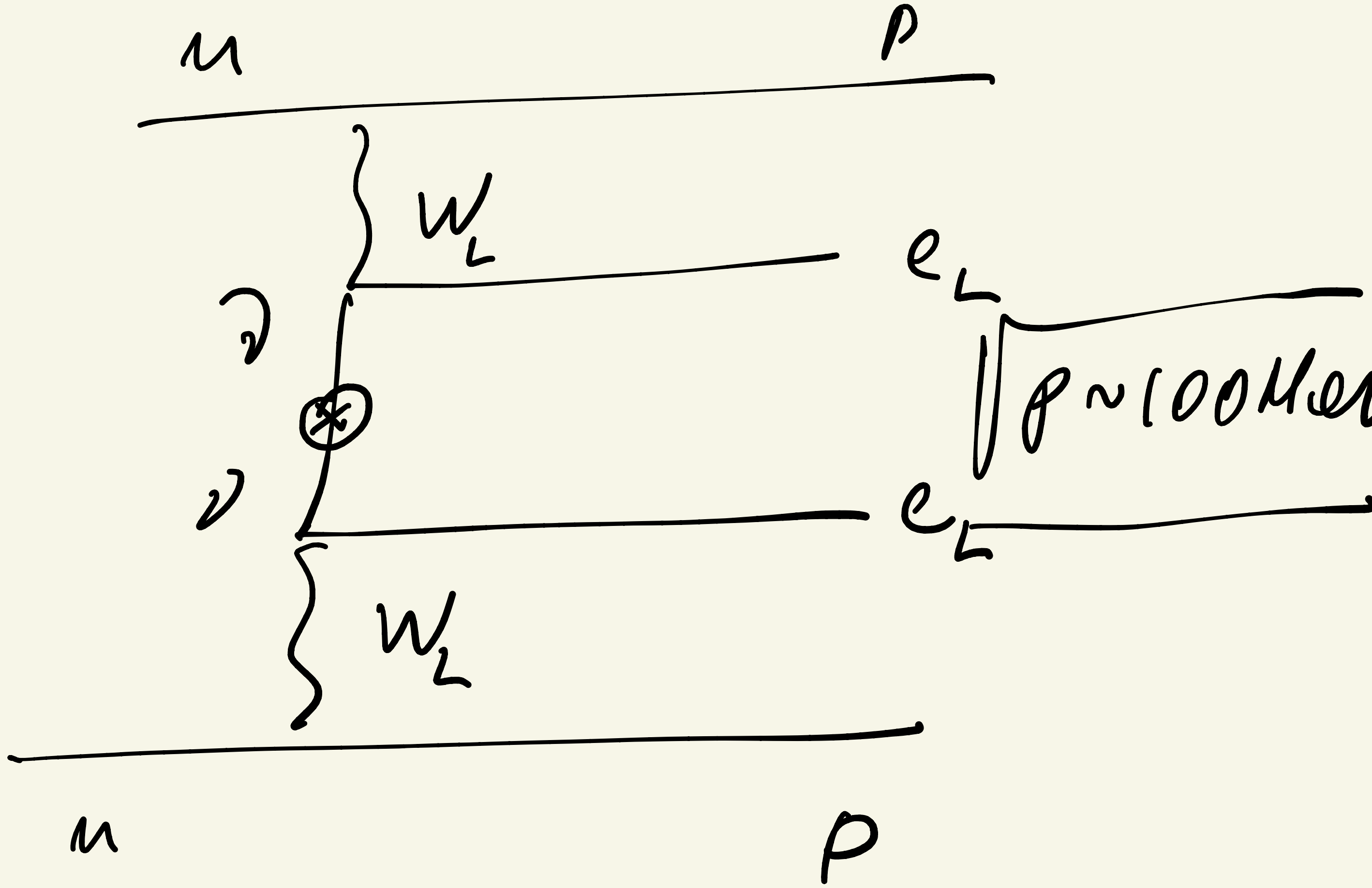
$$N \to e_L + W_L \quad (A \nu N)$$

$$\sqrt{\frac{1 \cdot M_\nu}{M_N}}$$

$$\boxed{\Gamma(N \to e\, W^+) = \Gamma(N \to \bar{e} + W^-)}$$

Ferrari et al. 2000

Hou et.al. 2017



$$\boxed{\begin{array}{c} W_L \leftrightarrow W_R\,, \quad \nu \leftrightarrow N \\[1em] e_L \leftrightarrow e_R \end{array}}$$

$\boxed{\delta^{++}_{R,L}}$ $\qquad \delta^{+}_{L}, \quad - - \; -$

$\delta^{--} \longrightarrow ee, \; e\mu$

$\mu\tau, \; e\tau$

$\tau\tau$

$l_R^{T} C \, y_{\Delta R} \, \Delta_R \, l_R$

δ_R^{++}

$\boxed{M_N \propto y_{\Delta R}^{*}}$