BBSM Neutrino Course

Lecture XXII

LMU

Spring 2020

Domain Walls

$$\Delta_L \xleftrightarrow[P]{} \Delta_R$$

$$\boxed{P^2 = 1}$$

$$V = -\frac{\mu^2}{2}\left(\Delta_L^2 + \Delta_R^2\right) + \lambda\left(\Delta_L^2 + \Delta_R^2\right)^2$$

$$+ \boxed{\lambda' \Delta_L^2 \Delta_R^2}$$

$\underline{\lambda' > 0}$ $\Rightarrow \langle \Delta_L \rangle = 0 \,,\ \langle \Delta_R \rangle \neq 0$

$$\boxed{\Delta = \text{triplets under } SU(2)}$$

$\langle \Delta_L \rangle = 0 \longrightarrow$ concentrate on Δ_R

$$V = -\frac{\mu^2}{2} \mathrm{Tr}\, \Delta_R^\dagger \Delta_R + \lambda_1 \left(\mathrm{Tr}\, \Delta_R^\dagger \Delta_R\right)^2$$

$$+ \lambda_2\, \mathrm{Tr}\, \Delta^2\, \mathrm{Tr}\, \Delta^{\dagger 2}$$

$$\mathrm{Tr}\, \Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R = f(\quad)$$

$\mathrm{Tr}\, \Delta_R^\dagger \Delta_R =$ does not care who gets vev

$\Downarrow$

$\Delta_R = 3$ complex $= 6$ real

$\Downarrow$

$SO(6)$ symmetry

$\lambda_2 = 0$: $SO(6) \longrightarrow SO(5)$

$$\Rightarrow \frac{6 \cdot 5}{2} - \frac{5 \cdot 4}{2} = 5$$

<u>broken gen.</u>

$\Rightarrow$ 5 NG bosons

3 are eaten : W_R^+, W_R^-, Z_R

$\Downarrow$

2 NG

· $\lambda_2 \neq 0$, $\boxed{\lambda_2 > 0}$

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$ local minimum

$\delta_R^{\pm\pm}$, $\delta_R^{\pm}$, $\delta_R^0 = v + h + i\, G_R$

$\delta_R^{\pm}$ → eaten by $W_R^{\pm}$

G_R ← eaten by Z_R

$$\boxed{m_\delta^{2\,++} \propto \lambda_2 v_R^2}$$

$\boxed{\lambda_2 = 0 \Rightarrow 2 \text{ NG bosons}}$

$SU(2)_R \times U(1)_{B-L}$

gauge ↓ $4 - 1 = 3$ broken

$U(1)_Y$ ⇓

3 massive $W_R^{\pm}, Z_R$

go back to SM

$SU(2) \times U(1)$ gauged

$$V \Rightarrow SO(4)_{\text{global}} \cong \underbrace{SU(2)_L}_{\text{gauged}} \times \underbrace{SU(2)_R}_{\text{part gauged}}$$

- gauge: $SU(2) \times U(1) \to U(1)$

  $3 + 1 - 1 = 3$

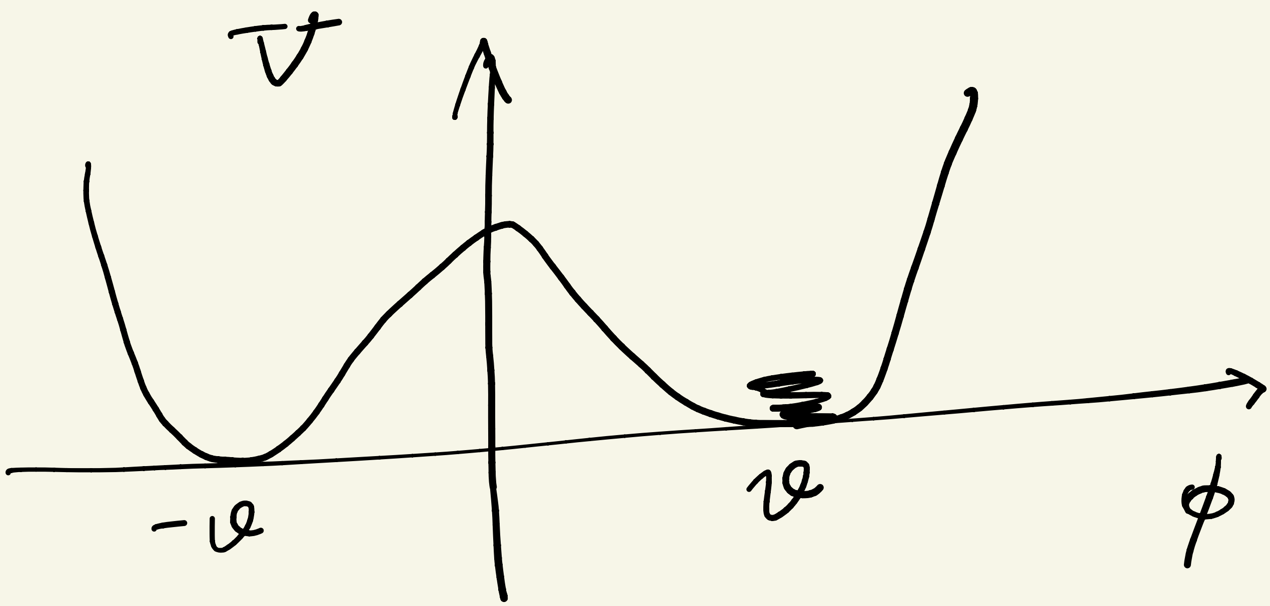
- global $SO(4) \to SO(3)$

  $6 - 3 = 3$

$$\boxed{D = \mathbb{Z}_2 \quad \text{discrete}}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi)$$

$\phi \in \mathbb{R}$ $\qquad V = \frac{1}{2}(\phi^2 - v^2)^2$

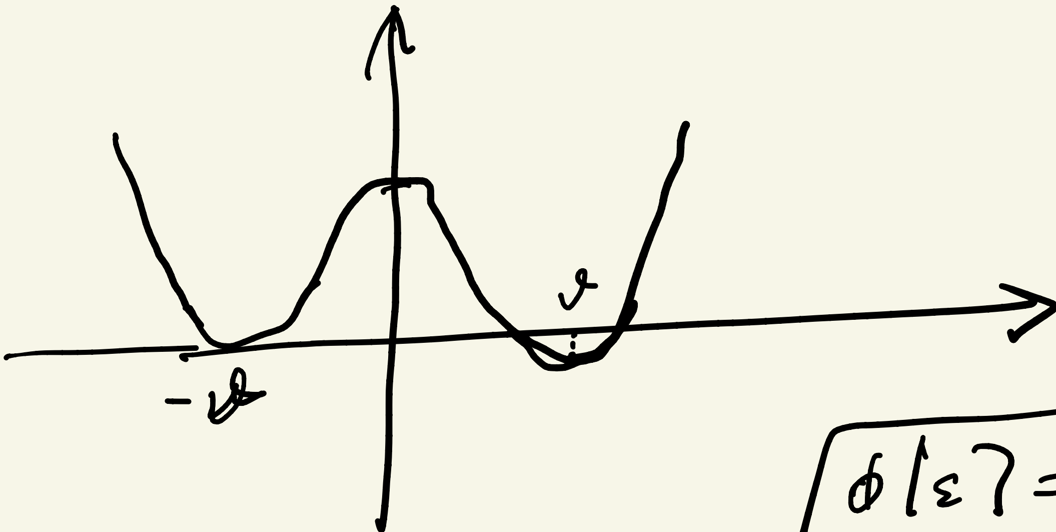


NO symm. breaking in QM

= tunneling

QM: $$\boxed{\Gamma \propto e^{-\int_a^b dx \sqrt{V}}}$$

Coleman '74



$\boxed{\phi\,[\varepsilon] = [m]^4}$

$$V = (\phi^2 - v^2)^2 + \frac{\varepsilon}{v}(\phi - v)$$

$$\Gamma_{tun} \propto e^{-v^{12}/\varepsilon^3}$$

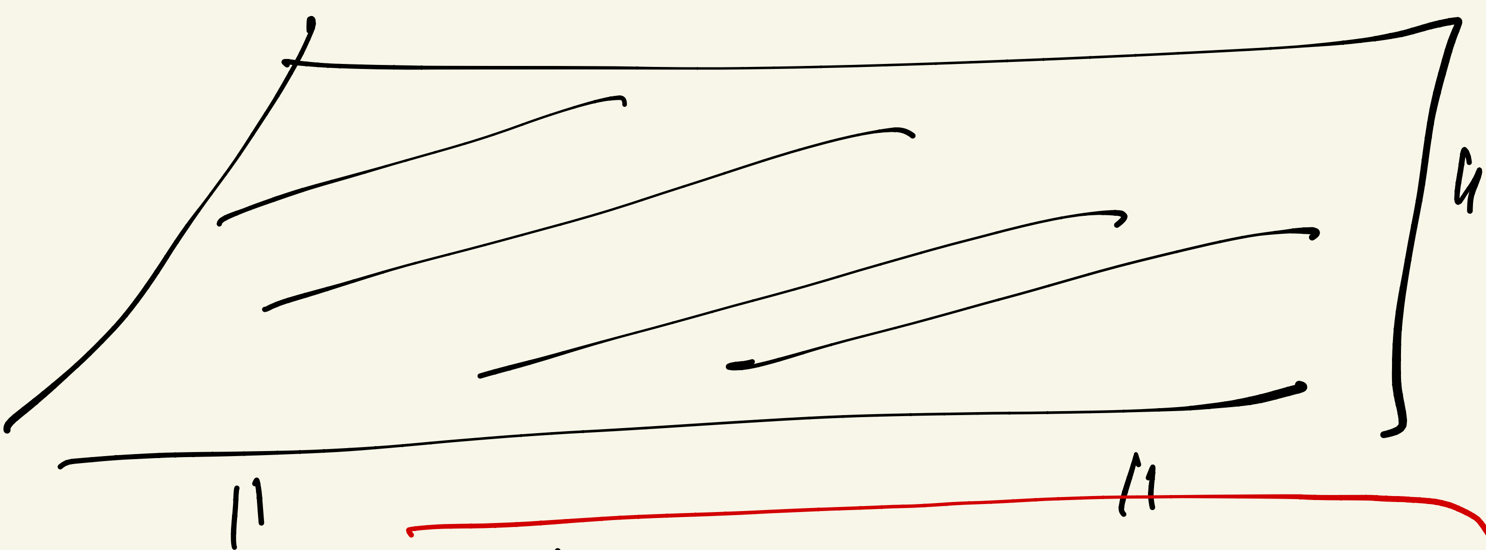
$$\mathcal{M}_0 = \{\phi_0 : V = \min\} = \{\phi_0^2 = v^2\}$$

$$= \{\pm v\} = \mathbb{Z}_2$$

vacuum manifold

$$\underbrace{\underbrace{\phi_0^{G} = v, \quad \phi_0^{M} = -v}_{V_0 = 0, \quad K_E = 0}}_{E = 0 \text{ ground states}}$$

$\phi_{cl} = \phi_{dw} = \phi_w =$ nontrivial f-n of z



[$S \to \infty$ wall $(x\text{-}y)$

$\phi_w(z)$ static solution]

$$\boxed{\text{finite energy (density)}}$$

$$\underbrace{E/S}_{\text{finite}} = \int_{-\infty}^{+\infty} dz \left[\frac{1}{2}\left(\frac{d\phi}{dz}\right)^2 + V \right]$$

$$\left(\frac{d\phi}{dz}\right)^2 \to 0, \quad V \to 0$$

at ∞

$$\boxed{\mathcal{M}_\infty = \{ z = +\infty, -\infty \}}$$

$$\boxed{\mathcal{M}_0 = \{ +v, -v \}}$$

$$\left.\begin{aligned} \mathcal{M}_\infty :\quad & V \to 0 \\ & \frac{d\phi}{dx} \to 0 \end{aligned}\right\} \Rightarrow \boxed{\phi_a \Rightarrow \pm v}$$

$$\boxed{\text{map from } \mathcal{M}_\infty \text{ to } \mathcal{M}_0}$$

(1) <u>trivial map</u>

$$\phi_0(z=+\infty) = \phi_0(z=-\infty) = +v$$

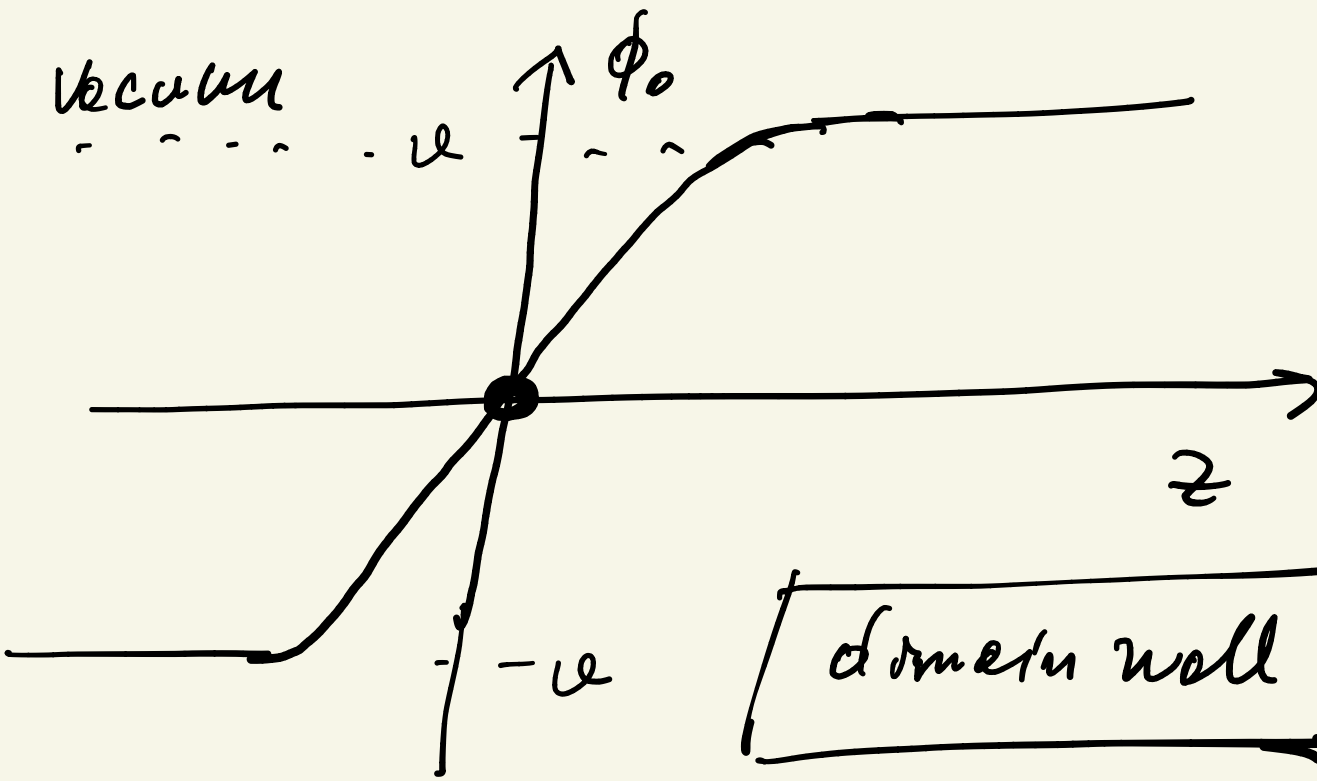
$$\mathcal{M}_\infty \to +v$$

solution = vacuum

(ii) non-trivial map

(a) $\phi_0(z=+\infty) = +v$

$\phi_0(z=-\infty) = -v$



$\boxed{\text{domain wall}}$

(b)

$$\left.\begin{aligned} \phi_0(z=+\infty) &= -v \\ \phi_0(z=-\infty) &= +v \end{aligned}\right\} \boxed{\text{anti wall}}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$$

$$\Box \phi = - \frac{\partial V}{\partial \Phi} \qquad (1)$$

static

$$\Downarrow$$

$$\frac{d^2 \phi_w}{dz} = \frac{\partial V}{\partial \phi} \cdot \frac{d\phi_w}{dz}$$

$$\Downarrow$$

$$\boxed{\frac{1}{2} \frac{d}{dz}\left(\frac{d\phi_w}{dz}\right)^2 = \frac{dV}{dz} \qquad (2)}$$

$$\Downarrow$$

$$\frac{1}{2}\left(\frac{d\phi_w}{dz}\right)^2 = V + \text{const.} \qquad (3)$$

at ∞: $\quad 0 \quad\quad 0 \quad\quad$ finite energy

$\Rightarrow$ Const. $= 0$

$\Downarrow$

$$\boxed{\frac{d\phi_w}{dz} = \pm\sqrt{2V}} \quad (4)$$

$$V = \frac{1}{2}(\phi^2 - v^2)^2$$

$\Downarrow$

$$\boxed{\frac{d\phi_w}{dz} = \pm(\phi_w^2 - v^2)}$$

$\Downarrow$

$$\frac{d\phi_w}{\phi_w^2 - v^2} = \pm\, dz \qquad (5)$$

$\Downarrow$

$$\left.\begin{array}{l}\boxed{\phi_W = v \tanh vz} \\[1em] \boxed{\phi_{\text{anti }W} = -v \tanh vz}\end{array}\right\} (6)$$

thickness $\boxed{\delta_W = \frac{1}{v}}$

$$E/S = \int_{-\infty}^{+\infty} dz \left[\frac{1}{2}\left(\frac{d\phi_W}{dz}\right)^2 + \overline{V} \right]$$

$$= \int_{-v}^{v} d\phi \, \frac{dz}{d\phi} \, 2\overline{V} = \pm\int_{-v}^{+v} d\phi \sqrt{2\overline{V}}$$

$$\left(\frac{d\phi}{dz} = \sqrt{2V} \right)$$

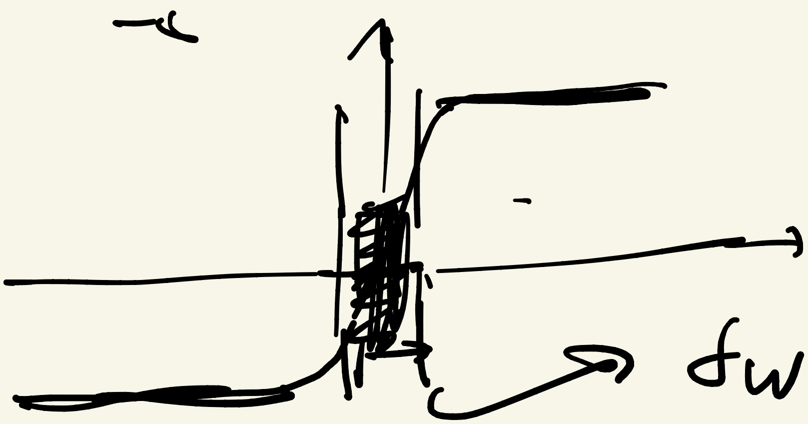
$$\boxed{E/S \simeq \pm \int_{-v}^{+v} d\phi\,(\phi^2 - v^2) \simeq \pm v^3}$$

$$\boxed{E \sim v^3 S}$$

Energetics

$$E/S = \int_{-\infty}^{+\infty} dz \left[\frac{1}{2}\left(\frac{d\phi}{dz}\right)^2 + V(\phi) \right]$$

*



$\downarrow$

$$V(\phi) = \frac{1}{2}(\phi^2 - v^4)^2 = \frac{1}{2} v^4 \text{ at } \phi = 0$$

$\boxed{V(\phi) \text{ piece}}$ $\quad \frac{1}{2} v^4 \cdot \int_{-\delta}^{\delta} = v^4 \delta$

* $\boxed{\text{minimum} \rightarrow \delta \rightarrow 0}$

$\boxed{\text{gradient}}$ $\quad \frac{1}{2} \left(\frac{\Delta \phi_w}{\Delta z} \right)^2 \cdot 2\delta$

$$= \frac{1}{2} \left(\frac{2v}{2\delta} \right)^2 2\delta = \frac{v^2}{\delta}$$

$$E/_S = v^4 \delta + v^2/_\delta$$

$$\frac{\partial V}{\partial \delta} = 0 \Rightarrow v^4 = v^2/_{\delta^2}$$

$$\delta = \frac{1}{2e}$$

$$j_\mu = \epsilon_{\mu\nu} \partial^\nu \phi \qquad \mu, \nu = t, z$$

$$\epsilon_{\mu\nu} \equiv - \epsilon_{\nu\mu}$$

$$\partial^\mu j_\mu = 0 = \epsilon_{\mu\nu} \partial^\mu \partial^\nu \phi$$

$$j_0 = \frac{d\phi}{dz} \Rightarrow$$

$$Q = \int_{-\infty}^{+\infty} dz \, \frac{d\phi}{dz}$$

$$Q = \phi(+\infty) - \phi(-\infty)$$

$$\frac{dQ}{dt} = 0$$

$Q(\text{vacuum}) = 0$;

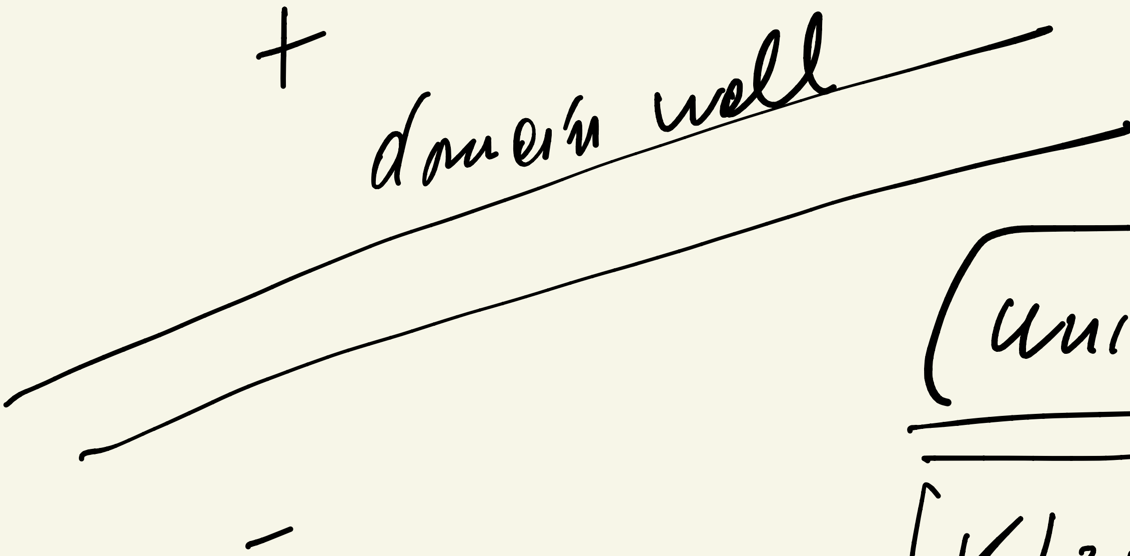
$Q(\text{wall}) = 2\vartheta$;

$Q(\text{anti-wall}) = -2\vartheta$

$\Downarrow$

Wall is stable!

· wall + anti-wall $\longrightarrow$ vacuum



universe

Kobzarev, Okun, Zeldovich '70

$$\boxed{E/S = v^3}$$

$$\boxed{v \gtrsim \text{TeV}}$$

$$\boxed{E_{\text{visible}} \simeq 1/10\, E_{\text{universe}}}$$

$$E_{\text{univ}} \simeq 10\, E_{\text{visible}}$$

baryons

$$\boxed{\frac{n_B}{n_\gamma} = 10^{-10}}$$

radiation

$$\boxed{E_\gamma \simeq {}^\circ K \simeq 10^{-4}\, eV}$$

$$E_B \simeq GeV \simeq 10^{9}\, eV \simeq 10^{13}\, E_\gamma$$

$$E_{univ} = 10\, E_B = 10\, N_B\, E_B$$

$$= 10 \cdot 10^{-10}\, N_\gamma\ GeV$$

$$= 10 \cdot 10^{-10} (R^3 \cdot n_\gamma)\ GeV$$

$$\boxed{E_{cmv} = 10^{-9} (RT)^3 \text{ GeV}}$$

$$\boxed{E_{well} = v^3 R^2}$$

$T = 10^{-4}$ eV

$T = 10^{-13}$ GeV

$R = 10^{28}$ cm

$RT = 10^{28} \cdot 10^{-13}$ cm GeV

ⓔ proton

$$\boxed{\text{GeV}^{-1} = 10^{-14} \text{ cm} \Rightarrow \text{GeV cm} \simeq 10^{14}}$$

$$\boxed{RT = 10^{29}}$$

$$\Downarrow \quad \boxed{RT = \text{const.}}$$

$$\frac{E_{well}}{E_{univ}} \simeq \frac{v^3 R^2}{10^{-9} (R\,T)^3 \,\mathrm{GeV}} \simeq$$

$$\gtrsim \frac{10^{9}\,\mathrm{GeV}^3\, R^2}{10^{-9} \cdot 10^{87}\,\mathrm{GeV}}$$

$$\gtrsim 10^{-69} (\mathrm{GeV}\, R)^2$$

$$\gtrsim 10^{-69} (\mathrm{GeV}\; 10^{28}\,\mathrm{cm})^2$$

$$\gtrsim 10^{-69} (10^{42})^2 \gtrsim \underline{\underline{10^{15}}} \;!!!$$

Standard cosmological model

$\Downarrow$

big bang

1. cold today
2. hot yesterday
3. isotropic and homogeneous

$d_H = t$

$$\boxed{t \simeq \frac{1}{T}\frac{M_{pl}}{T}}$$ ← gravity

$\Downarrow$

Last

$$R(T) = \frac{c}{T} \qquad c = 10^{29}$$

$$d_H(T) = \frac{M_{pe}}{T^2}$$

size

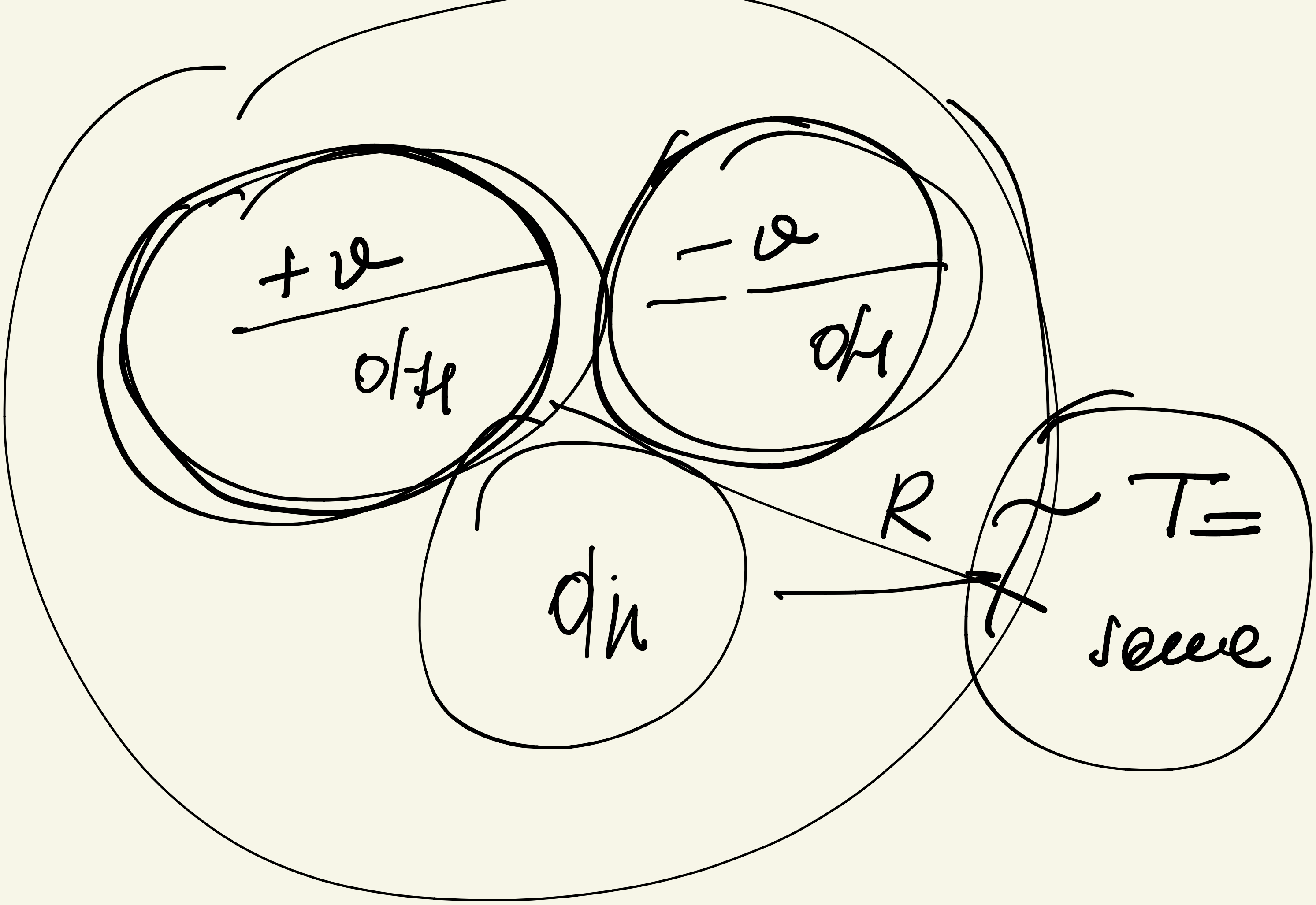
$$\Downarrow$$

$$\boxed{\frac{R(T)}{d_H(T)} = c\,T/M_{pe}}$$

causal contact

$$\frac{R_0}{d_{H^0}} = 1 \quad (\text{today})$$

$$\boxed{R(T) \gtrsim d_H(T)}$$



$c = MeV = 10^{-3} GeV$

$$\boxed{R/d_n \simeq 10^{29} \cdot \frac{10^{-3}}{10^{+18}} = 10^{8}}$$