BBSM Neutrino Course

Lecture XVI

LMU

Spring 2020

It's neutrino, stupid!

SM in a nutshell

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad\qquad u_R$$

$$d_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad\qquad e_R$$

doublets $SU(2)$ singlets $SU(2)$

Φ – doublet, $Y = 1$

$$\Phi \longrightarrow U \Phi \qquad U^\dagger U = 1$$

$$Q = T_3 + \frac{Y}{2} = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} + \frac{1}{2}\mathbb{1}$$

$$\Rightarrow \quad \Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \begin{matrix} \text{charged} \\ \text{neutral} \end{matrix}$$

$$\Phi = \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$ Higgs boson

v vev = vacuum expectation value

v = const., $d = [\text{mass}]$

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v → masses

$\Rightarrow \quad m_f = y_f\, v$ — $(\bar{u}\,\bar{d})_L\, Y_d\, \Phi\, d_R$

Yukawa

$$\boxed{M_A = 0, \quad M_W = \frac{g}{2} v, \quad M_Z \cos\theta_W = M_W}$$

$$\mathcal{L} = \frac{1}{2}(D_\mu \Phi)^\dagger (D^\mu \Phi) - V$$

$$\Phi \to U\Phi$$

$$U = e^{i T_a \theta_a(x)}$$

$$Y=1 \Rightarrow \Phi \to e^{i\chi}\,\Phi$$

$$U_Y(1)$$

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} R_1 + iR_2 \\ R_3 + iR_4 \end{pmatrix} \quad (1)$$

$$= \begin{pmatrix} G_2/2 - iG_1/2 \\ v + h - iG_3/2 \end{pmatrix} \quad (2)$$

$$= e^{iG_i/v\, T_i} \begin{pmatrix} 0 \\ v+h \end{pmatrix} \quad \text{(phase)} \quad G1$$

dimensions

$$T_a = \frac{\sigma_a}{2}$$

$$\simeq \left(1 + i\, G_i/v\ \sigma_i/2 + \dots\right) \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \left[1 + i \begin{pmatrix} G_3/2v & (G_1 - iG_2)/2v \\ (G_1 + iG_2)/2v & -G_3/2v \end{pmatrix}\right] \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \begin{pmatrix} (G_2 - iG_1)/2 \\ v + h - iG_3/2v \end{pmatrix} + \dots \quad (2)$$

$$\boxed{\Phi = e^{i G_i \sigma_i / 2v} \begin{pmatrix} 0 \\ v+h \end{pmatrix}}$$

How to write interactions?

- respect the symmetry

$$\boxed{SU(2) \quad \overset{\text{rench}}{r} = 1}$$

- $d[V] \leq 4$

$$\to \quad \Phi^+ \Phi \to \Phi^+ U^+ U \Phi$$

$$+ (\Phi^+ \Phi)^2 \qquad = \Phi^+ \Phi$$

$$\cancel{+ (\Phi^+ \Phi)^3 \frac{1}{\Lambda^2}} \longleftrightarrow \text{Fermi}$$

$d[\Phi] = 1$ (mass)

$\Downarrow$

$$V(\Phi) = \frac{\lambda}{4}\left(\Phi^\dagger \Phi \overset{\text{crux}}{-} v^2\right)^2$$

$$= \frac{\lambda}{4}(\Phi^\dagger\Phi)^2 \underbrace{- \frac{\lambda}{2} v^2 \Phi^\dagger \Phi}_{\frac{\mu^2}{2}} + \frac{\lambda}{4} v^4$$

$\boxed{\text{minimum}}$

vacuum manifold $\mathcal{M}_0 \Rightarrow$

$$\mathcal{M}_0 = \{ V = V_{min} = 0 \} = \{ \Phi_0^\dagger \Phi_0 = v^2 \}$$

$$\Phi = \begin{pmatrix} R_1 + iR_2 \\ R_3 + iR_4 \end{pmatrix} \Rightarrow \Phi^\dagger \Phi = \sum_{i=1}^{4} R_i^2$$

$$\mathcal{M}_0 = \{ R_i : \sum_{i=1}^{4} R_i^2 = v^2 \} = S_3$$

equivalence of S_3 points

$\underline{U(1)}$: $\left[R_{10}^2 + R_{20}^2 = v^2 \Longrightarrow \boxed{\mathcal{M}_0 = S_1} \right]$

$$\phi = R_1 + i R_2$$

$\underline{SO(3)}$: $\Phi = \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} \Rightarrow \underline{R_{10}^2 + R_{20}^2 + R_{30}^2 = v^2}$

$$\boxed{\mathcal{M}_0 = S_2}$$

$$\Phi_0^{Gowu} = \begin{pmatrix} 0 \\ v \end{pmatrix} \Longleftrightarrow \Phi = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

$$\boxed{\text{only neutral fields can have vev} \neq 0}$$

$$\Phi_0^{Max} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

charged field has a vev ???

$$\Phi_0^G = \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad T_a \Phi_0^G \neq 0$$

$(Y=1)$

$$Y \Phi_0^G \neq 0$$

$$\Rightarrow \quad Q_G \Phi_0^G = \left(T_3 + Y/2\right) \Phi_0^G =$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Phi_0^G = 0$$

but $Q_G \Phi_0^M = \Phi_0^M \qquad \Phi_0^M = \begin{pmatrix} v \\ 0 \end{pmatrix}$

$$Q_M = T_3 - \frac{Y}{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \boxed{Q_M \Phi_0^M = 0}$$

In both cases: massless photon

$$\Phi^{0\,to} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad v^2 = v_1^2 + v_2^2$$

$$Q_G\, \Phi_0^0 \neq 0, \quad Q_M\, \Phi_0^0 \neq 0$$

$\boxed{\text{Massive photon?!}}$

$$Q_{ob} = a T_3 + b \frac{Y}{2}$$

$$= \begin{pmatrix} a+b & \\ & -a+b \end{pmatrix} \frac{1}{2}$$

$$Q_{ofo}\, \Phi^{ofo} = \begin{pmatrix} (a+b)\,v \\ (-a+b)\,v_2 \end{pmatrix} \frac{1}{2} = 0$$

$$a+b=0, \quad -a+b=0$$

impossible!!

$\boxed{\text{Prove!}}$ $\boxed{Q_{oto} = c_a T_a + d\, Y/2}$ $\therefore Q_{oto}\, \Phi_0^{oto} = 0$

$$V = \frac{\lambda}{4}\left(\Phi^{+}\Phi - v^2\right)^2$$

$$V_{min} = 0$$

physics does not depend on the value of V_{min} (ground state)

WRONG!

$\boxed{\text{gravity knows about } V_{min}}$

$\boxed{\text{cosmological constant} \propto \text{value of } V_{min}}$

⊗ $\Phi = e^{iG_i/v\, T_i}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$ phase rotation

$\Downarrow$

$\Phi^{+}\Phi = (v+h)^2$ $\boxed{\text{no } G \text{ in } V}$

Gravity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

energy - momentum

$$\Rightarrow \boxed{T_{\mu\nu} = f(\mathcal{L}) = f(v)}$$

$$\boxed{\Phi^+\Phi = (v+h)^2 \Rightarrow m_{G_i} = 0}$$

3 massless particles

$$\left.\begin{array}{l} T_i \Phi_0 \neq 0, \quad Y \Phi_0 \neq 0 \\ Q \Phi_0 = 0 \end{array}\right\} \text{3 "broken" generators}$$

$$\boxed{\text{"broken" generator } T_a \Longleftrightarrow T_a \Phi_0 \neq 0}$$

Goldstone theorem:

$$T_a \Phi_0 \neq 0 \Longleftrightarrow m_{G_a} = 0$$

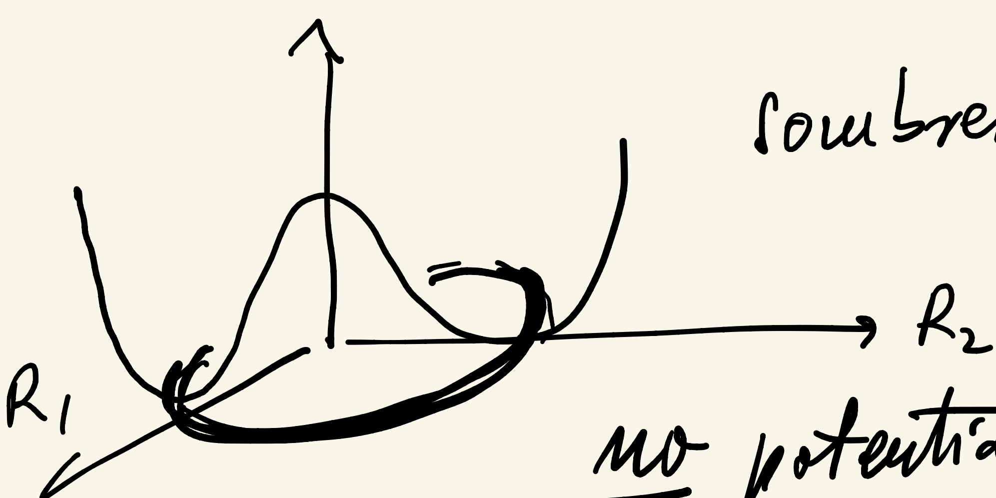
G_a = Goldstone

(Nambu-Goldstone)

$\boxed{U(1)}$ $$V = \frac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$

$\phi \in \mathbb{C}$ $$\mathcal{M}_0 = S_1$$

$$\phi = R_1 + iR_2$$



sombrero potential

<u>no</u> potential energy

$$\Downarrow$$

massless field

$$\boxed{\phi = \underbrace{v}_{\text{vev}} + h + i\sigma} = e^{i\sigma'/v}(v+h)$$

$$\Downarrow$$

$$\underline{m_{\sigma'} = 0}$$

$$\phi^{*}\phi = (v+h)^2 + \sigma^2$$

$$\Rightarrow \quad V = \frac{\lambda}{4}\left[(v+h)^2 + \sigma^2 - v^2\right]^2$$

$$= \frac{\lambda}{4}\left[2vh + h^2 + \sigma^2\right]^2$$

$$= \frac{\lambda}{4}(h^2+\sigma^2)^2 + \frac{1}{2}(2\lambda v^2)\, h^2 +$$

$$+ \lambda v h (h^2 + \sigma^2)$$

$$\boxed{m_\sigma = 0}$$

U(1) GTM

$$\phi = e^{iG/v}(v+h)$$

$$\boxed{\phi^* \phi = (v+h)^2 \Rightarrow m_G = 0}$$

$$\frac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi) \longrightarrow \text{messy}$$

$$\hookrightarrow \left(\frac{\partial_\mu G}{v}\right) \cdots$$

· derivative , $(1/v)$

SM

$$\Phi = e^{iG_i T_i/v}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

G_i = massless NG bosons

right??

WRONG!!!

$$\Phi \to e^{i\theta_i(x) T_i}\, \Phi \quad \text{(gauge inv.)}$$

$$\theta_i = -G_i/v$$

$$\Rightarrow \boxed{\Phi \to \begin{pmatrix} 0 \\ v+h \end{pmatrix} = \Phi_{un}}$$

unitary "gauge"
physical gauge

I gauged away G_i!

Gi are gone

What happened ???

I lost 3 degrees of freedom!?

???

$M_W \neq 0 \neq M_Z$

massive Proca $= 3 (= 2+1)$ d.o.f.

- $\boxed{\text{started with}}$

$$\underbrace{A_i, B}_{\substack{4 \text{ massless gauge field} \\ (\text{each} = 2 \text{ d.o.f.})}}, \quad \Phi = 4 \text{ d.o.f.}$$

3 less

- $\boxed{\text{end up}}$

$$\underbrace{A\ (m_A = 0)}_{2 \text{ d.o.f.}}, \quad \underbrace{W, Z}_{3 \times 3 \text{ d.o.f.}} \quad + \quad h$$

$\boxed{3 \text{ more}}$

Higgs

1

gauge boson "eats" a Goldstone

U(1)

initially

$\phi \in \mathbb{C}$ (2)

A ($m_A = 0$) (2)

finally

h (1)

$m_A \neq 0$ (3)

$G \longrightarrow$ becomes a longitudinal component

$$\Phi = \begin{pmatrix} 0 \\ v + h \end{pmatrix} = \text{correct}$$

physical

Only one scalar boson =

= Higgs (Weinberg) boson

$$\boxed{\text{SM}}$$

$$\mathcal{L} = \frac{1}{2}(D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi)$$

$$+\sum_f i\bar{f}\gamma^\mu D_\mu f - \mathcal{L}_Y$$

$$-\underbrace{\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a}}_{SU(2)} - \underbrace{\frac{1}{4}B_{\mu\nu}B^{\mu\nu}}_{U(1)}$$

$$F^a_{\mu\nu} = \partial_\nu A^a_\mu - \partial_\mu A^a_\nu + g\,\epsilon_{abc}A^b_\mu A^c_\nu$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$\mathcal{L}_Y = (\bar{u}\,\bar{d})_L\, Y_d\, \Phi\, d_R + \cdots$$

$$\boxed{m_f = y_f v,} \qquad \boxed{M_W = g/2\, v}$$

$$V = \frac{\lambda}{4}(\Phi^\dagger\Phi - v^2)^2 = \frac{\lambda}{4}\left[(v+h)^2 - v^2\right]^2$$

$$= \frac{\lambda}{4}\left(2vh+h^2\right)^2 =$$

$$= \frac{\lambda}{4} h^4 + \lambda v h^3 + \frac{1}{2}\left(2\lambda v^2\right) h^2$$

$$\boxed{m_h^2 = 2\lambda v^2}$$

One resp: "you do not predict Higgs mass"

unlike W mass

weak
$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2} = \frac{e^2}{M_W^2 \sin^2\theta_W}$$

measure

$\Downarrow$

predict M_W!

$$J_\mu^Z = \frac{g}{\cos\theta_W}\left(T_3 - Q\sin^2\theta_W\right)$$

$$\Downarrow$$

$\boxed{\text{- W mass related to interaction (weak)}}$

$\Downarrow$ the same for the Higgs

$$\boxed{M_W = \frac{g}{2} v} \longleftrightarrow \boxed{g M_W W^+ W^- h}$$

$$m_h = \sqrt{2\lambda}\, \frac{2}{g} M_W$$

λh^4 — new int.

relate Higgs mass to new (weak) int.

· fermions

$$\frac{g}{2} \frac{m_f}{M_W} h \bar{f} f$$

$m_f \xleftrightarrow[\text{related}]{} $ Yukawa

$m_f \Rightarrow \Gamma(h \to f\bar{f})$

$\Downarrow$

3rd gen decided — τ, b, t + W, Z

• $\langle \bar{q}_L q_R \rangle \neq 0$ — • quark condensate

$\simeq \Lambda_{QCD}^3 \simeq (\mathrm{GeV})^3$

doublet

$\Lambda_{\text{quark}} \lesssim \mathrm{GeV}$

$\Downarrow$

W, t mass from $\langle \bar{q} q \rangle$

Why elementary scalar?

There was none

Weinberg, Susskind, - - -

"Techni Color"

$$\langle \bar{Q} Q \rangle = \Lambda_{TC}^{3}$$

$$\Lambda_{TC} \simeq M_W$$

SM: $V = \frac{\lambda}{4} \left(\Phi^+ \Phi - v^2 \right)^2 \quad (d \leq 4)$

negative

$\longrightarrow -\frac{\lambda}{2} v^2 \Phi^+ \Phi$ tachyon?

$\Phi = \begin{pmatrix} 0 \\ v + h \end{pmatrix} \longrightarrow m_h^2 = \lambda v^2 (2)$

$\boxed{\text{"uly – sign" ?????}}$

Uly + sign ?