

BB SM Neutrino Course

Lecture XIV

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LMU

Spring 2020

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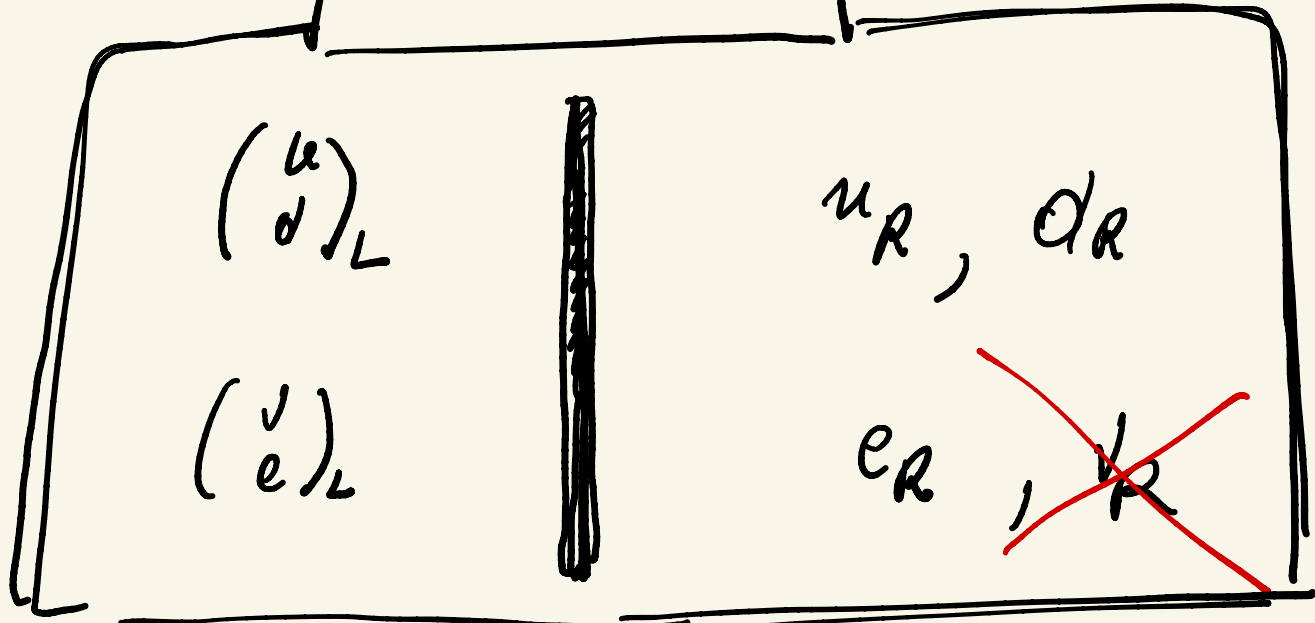
It's neutrino, stupid!

Towards a theory of  
neutrino mass



SM perspective

$P$  violation



$10^{-5} \text{ GeV}^{-2}$

$$SU(2)_L \times U(1)$$

$$Q = T_3 + \frac{Y}{2}$$

electro-weak theory

Feynman

$G_F$

$$J_W \bar{J}^W$$

effective theory

'50 A

QED

$$e J_{em}^\mu A_\mu \quad \frac{e^2}{4\pi} = \alpha_{em}$$

messengers

weak int  
is  
messengers

Glashow '1961

$$g W_\mu J_W^\mu \rightarrow \frac{g^2}{4\pi} = \alpha_W$$

Minimal gauge theory  
of weak int

$$SU(2) = 3 \text{ gen.}$$

↓ can it work?

$$D_\mu = \not{\partial}_\mu - ig T_a A_\mu^a$$

$$T_a \quad a = 1, 2, 3$$

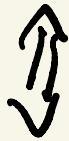
$$\frac{A_1 \mp i A_2}{\sqrt{2}} = W^\pm$$

$$T_3 \leftrightarrow A_3 \stackrel{?}{=} A \text{ photon}$$

why A is not photon?



(i)  $W^\pm (A_1, A_2)$  — pure LH



$A_3 \rightarrow$  LH fermions

$$A \Rightarrow L + R$$

(ii)

$$T_3 = \frac{Q_3}{2} \quad \text{quantized}$$



$$Q_{em} = T_3 = \text{quantized}$$

$$q = u \frac{1}{2}$$

$$q_u = \frac{2}{3}, \quad q_d = -\frac{1}{3}$$

$$q_\nu = 0, \quad q_e = -1$$

$SU(2) \times U(1)$

$$[Y, T_a] = 0$$

$$Y = 2(Q - T_3)$$

sacred

You cannot break it!

~~$m_d \bar{d}_L d_R$~~

$\hookrightarrow$  no  $d_L \rightarrow \begin{pmatrix} u \\ d \end{pmatrix}_L \equiv q_L$

$d=3$   $\left| \begin{matrix} y_d (\bar{u} \bar{d})_L \Phi d_R \end{matrix} \right.$

breaks  $SU(2)$

mass

$\hookrightarrow d=4$

scalar field  $\left. \begin{matrix} \cdot \text{dim. org} \\ \cdot \text{Lorentz} \end{matrix} \right\}$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow U \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$U^\dagger U = 1$$

- $\bar{\Phi} \rightarrow U \bar{\Phi}$  Weinberg '67  
Higgs doublet

- $Y \bar{\Phi} = ?$

$$Q = T_3 + \frac{Y}{2} \Rightarrow d_R: -\frac{1}{3} = \frac{Y}{2}$$

$$\Rightarrow Y_{d_R} = -\frac{2}{3}$$

$$d_L: -\frac{1}{3} = -\frac{1}{2} + \frac{Y}{2} \Rightarrow Y_{d_L} = \frac{1}{3}$$

$$(\bar{u} \bar{d})_L \bar{\Phi} d_R$$

$$Y: \quad -\frac{1}{3} \quad \textcircled{41} \quad -\frac{2}{3}$$

$$Q \bar{\Phi} = \left[ \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \right] \bar{\Phi}$$

$$\Downarrow = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \bar{\Phi}$$

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \text{ — neutral } (Q_{em})$$

$$\Downarrow \quad T_3 \varphi^0 = -\frac{1}{2} \varphi^0$$

$$\gamma d (\bar{d}_L \varphi^0 d_R + \bar{d}_R \varphi^{0*} d_L)$$

$\Downarrow$  mass for  $d$

$$\varphi^0 = \underbrace{(\vartheta)}_{\text{constant}} + h$$

constant

Higgs field

$$\Phi \rightarrow U \Phi$$

$$\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

ground state = vacuum

Vacuum  $\neq 0$

$$U \Phi_0 \neq \Phi_0 \therefore T_a \Phi_0 \neq 0$$



$$Y_d \underbrace{(\bar{d}_L d_R + \bar{d}_R d_L)}_{\bar{d}d} (\nu + h)$$

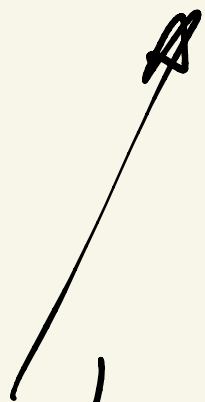
$$= Y_d \bar{d}d (\nu + h) \Rightarrow \boxed{m_d = Y_d \nu}$$

$$= m_d \bar{d}d + \boxed{\frac{m_d}{\nu} h \bar{d}d}$$

↗  
Feyn



Higgs couples to the mass of  $d$  quark



$$h \rightarrow \bar{d}d \Rightarrow \Gamma(h \rightarrow \bar{d}d) =$$

$$= \left(\frac{m_d}{\nu}\right)^2 \frac{m_d}{8\pi}$$

(ν = ?)

a quark mass?

$H^1_2$  hier, stay id?

$$\Psi_u (\bar{u}_L \bar{d}_L) \in \Phi^* u_R \quad ??$$

$Y:$

$$-\frac{1}{3}$$

$$\frac{4}{3}$$

$$Y_\Phi = +1$$

anti-doublet

anti-doublet

$$E \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = i\sigma_2$$

$\Downarrow$

$$\Psi_u (\bar{u}_L \bar{d}_L) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ h+u \end{pmatrix} u_R + h.c$$

$$= \gamma_u (\bar{\nu}_L \bar{d}_L) \begin{pmatrix} h + a \\ 0 \end{pmatrix} \nu_R + h.c.$$

$$= \gamma_u (\bar{\nu}_L \nu_R + \bar{u}_R d_L) (h + a)$$

$\Downarrow$

$$= m_u \bar{\nu} \nu + \frac{w_u}{e} \bar{u} \nu h$$

electron mass

→ couples to neutrinos

$$\gamma_e (\bar{\nu}_e)_L \not{D} e_R + h.c.$$

$\Downarrow$

$$m_e \bar{e} e + \frac{w_e}{e} h \bar{\nu} \nu$$

$\nu = ?$

$$\mathcal{L}_{\Phi} = \frac{1}{2} (D_{\mu} \Phi)^{\dagger} (D^{\mu} \Phi)$$

$U(1)$  coupling  
↗

$$\boxed{Y_{\Phi} = 1}$$

$$D_{\mu} = \partial_{\mu} - ig \underbrace{T_a A_{\mu}^a}_{SU(2)} - ig' \frac{Y}{2} B_{\mu}$$

$$T_a = \frac{\sigma_a}{2}$$

$SU(2)$  coupling  
↑

$U(1)$   
↑

$$\Phi = \begin{pmatrix} 0 \\ v+h \end{pmatrix} \rightarrow \text{Higgs}$$

int.

$$\cancel{D_{\mu} \Phi} = \cancel{\partial_{\mu} \Phi} - \frac{i}{2} \begin{pmatrix} g A_3 + g' B & g (A_1 - i A_2) \\ g (A_1 + i A_2) & -g A_3 + g' B \end{pmatrix} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

int

$$\rightarrow -\frac{i}{2} \begin{pmatrix} g (A_1 - i A_2) \\ (-g A_3 + g' B) \end{pmatrix} (v+h)$$



$$\frac{1}{2} (D_\mu \Phi)^\dagger (D^\mu \Phi) \rightarrow \text{'67 Weinberg}$$

$$\frac{1}{2} \frac{1}{4} \left[ g^2 (A_1^2 + A_2^2) + (g A_3 - g' B)^2 \right]$$

$\parallel$   
 W mass

$(v+h)^2$   
 $\uparrow$

$SU(2) \times U(1)$        $g$  &  $g'$

$$\int d^4x \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \bar{\psi} \gamma^\mu \partial_\mu \psi$$

$$Q_{em} = T_3 + \frac{Y}{2}$$

$$\int d^4x \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{g'}{g} \int d^4x \bar{\psi} \gamma^\mu \partial_\mu \psi$$

$$\tan \theta_W = g'/g$$

$$W_{\pm}^{\pm} = \frac{A_1 \mp i A_2}{\sqrt{2}} \quad \} \text{ (physical)}$$

$$\Rightarrow \frac{1}{2} \cdot \frac{1}{4} g^2 W_{\mu}^{\pm} W^{\mp \mu} (v^2 + 2v h + \dots)$$

$$\Rightarrow M_W = \frac{1}{2} g v$$

$$\tan \theta_w \equiv \frac{g'}{g}$$

$$\Rightarrow Z_{\mu} = \frac{(g A_3 - g' B)_{\mu}}{\sqrt{g^2 + g'^2}}$$

$$A_{\mu} \perp Z_{\mu} \equiv \frac{(g' A_3 + g B)_{\mu}}{\sqrt{g^2 + g'^2}}$$

⇓

$$A_{\mu} = \sin \theta_w A_{3\mu} + \cos \theta_w B_{\mu}$$

$$Z_{\mu} \equiv \cos \theta_w A_{3\mu} + \sin \theta_w B_{\mu}$$



$$\underbrace{M_A = 0}_{\text{photon!}}$$

$$M_z^2 = (g^2 + g'^2) v^2 / 4$$



$$M_z \cos \theta_w = M_w$$

bottom line

$\Phi =$  Higgs doublet

$$\Downarrow \Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

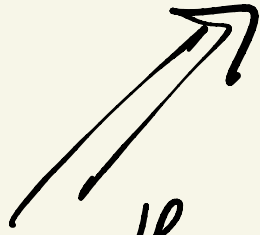
$$M_w = \frac{g}{2} v$$

arbitrary  $\rightarrow$



$$e = g' \sin \theta_w$$

$$\frac{m_f}{e} h \bar{f} f = g \frac{m_f}{M_W} \bar{f} f$$



theory of mass

$$m_f \Rightarrow \Gamma(h \rightarrow f \bar{f}) \propto m_f^2!$$

$$\left( g M_W W_\mu^+ W^{\mu-} + \frac{g}{\cos\theta_W} M_Z Z^\mu Z_\mu \right) h$$

show this!

$$\Phi_0 = \begin{pmatrix} 0 \\ \nu \end{pmatrix}$$

"crazy"  
vacuum

$$\left. \begin{array}{l} \cdot T_a \Phi_0 \neq 0 \\ \cdot Y \Phi_0 \neq 0 \end{array} \right\} Q_{em} = T_3 + Y/2$$



$$Q_{em} \Phi_0 = 0$$

worked  $\Leftrightarrow \not\neq$   
maximal

## Predictions

(i)  $\exists$  massless photon

(ii) massive  $Z$  boson  $M_Z = \frac{M_W}{\cos \theta_W}$

$$\theta_W \approx 30^\circ$$

$$M_W = 80 \text{ GeV}, M_Z = 90 \text{ GeV}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{e^2}{8M_W^2 \sin^2 \theta_W}$$

(iii) fermion Yukawa  $\propto \frac{m_f}{M_W}$

$$y_f = \frac{g}{2} \frac{m_f}{M_W}$$

$$y_f^2 \leq 4\pi \quad \alpha_y = \frac{y^2}{4\pi} \ll 1$$

perturbative

$$\Rightarrow \frac{m_f^2}{M_W^2} \ll \frac{4}{g^2} 4\pi$$

$$\boxed{m_f / M_W \ll 10}$$

"miracle"  $\left[ \begin{array}{l} \text{all fermion masses} \\ \leq M_W \end{array} \right]$

fermions live around  $M_W$

Imagine a LR symmetric world

$$\ell_L \equiv \begin{pmatrix} \psi \\ \psi \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} \psi \\ \psi \end{pmatrix}_R \equiv \ell_R$$

$SU(2) \times U(1)$

gauge  
weak int.  
theory



Mass terms  $\longleftrightarrow$  no wall

$$(\bar{u} \bar{d})_L \quad \textcircled{M} \quad \begin{pmatrix} u \\ d \end{pmatrix}_R = f m_i$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_{L,R} \rightarrow U \begin{pmatrix} u \\ d \end{pmatrix}_{L,R}$$

$$y_{2L} = y_{2R} = \frac{1}{3}$$

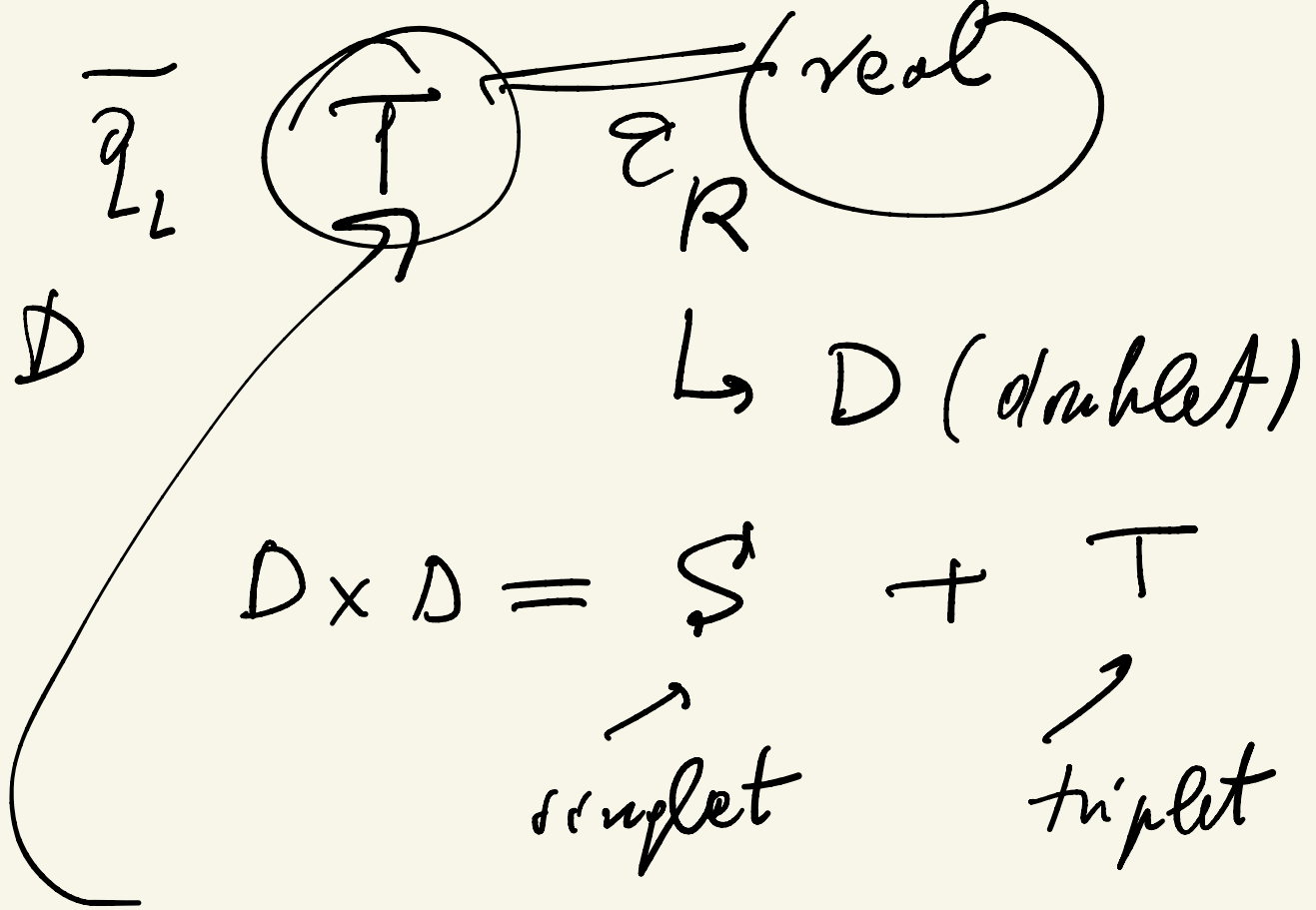
- $m_u = m_d$  wray!
- why  $m_f \approx M_w$ ? little "miracle"

miracles happen

- split  $M_u$  and  $M_d$







$T = \text{triplet} \equiv \text{adjoint rep.}$

$$T \rightarrow U T U^\dagger$$

$$T^\dagger \rightarrow U T^\dagger U^\dagger$$

$\Rightarrow T = T^\dagger$  irreducible

$$T \rightarrow U T U^\dagger = T \rightarrow U^\dagger U T = T$$

$$\Rightarrow \text{Tr } T = 0$$

$$T = T_a \psi_a \quad a=1,2,3$$

↑ triplet (vector)



$$y_{L_i} = y_{R_i} = 1/2$$

$$\mathcal{L}_Y = \bar{q}_L (M + Y_T T) q_R + h.c.$$

need  $T_0 \neq 0$

Higgs

$$\text{Tr } T = 0$$

$$\Rightarrow \text{Tr } T_0 = 0$$

$$\begin{aligned} \Rightarrow T_0 &= 2 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = 2\sigma_3 \\ &= 2\alpha T_3 \end{aligned}$$



$$(\bar{u} \bar{d})_L (\underline{M} + y \begin{pmatrix} \varrho_T & 0 \\ 0 & -\varrho_T \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}_R + \text{h.c.}$$



$$\begin{aligned} M_u &= \underline{M} + y \varrho_T \\ M_d &= \underline{M} - y \varrho_T \end{aligned}$$

$$T = (\varrho_T + h_T) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$T = SU(2)$  adjoint

$\Rightarrow y(T) = 0$  hypercharge



$$Q = \hat{T}_3 + Y/2$$

$$\hat{T}_3 = \text{generator}$$

$$SU(2) T_3$$

$$Q_{em} \mathbb{T} = \hat{T}_3 \mathbb{T}$$

$$Q_{em} T_0 = \hat{T}_3 T_0 = 0$$

$$T \rightarrow U T U^\dagger \quad U = e^{i T_a \theta_a}$$

$$= e^{i \sigma_a / 2 \theta_a}$$

$$= \left( 1 + i \sigma_a / 2 \theta_a \right) T \left( 1 - i \sigma_a / 2 \theta_a \right)$$

$$= T + i \theta_a \left[ \frac{\sigma_a}{2}, T \right]$$

$$\Rightarrow T_a \mathbb{T}_{field} = \left[ \frac{\sigma_a}{2}, \mathbb{T}_{field} \right]$$

$$T_0 = u \sigma_3$$

$$\Rightarrow T_3 T_0 = \left[ \frac{\sigma_3}{2}, \sigma_3 \right] u = 0$$



both  $T_3$  and  $\gamma$  are  
unbroken



two "photons" =  
= 2 massless gauge bosons



$$M_A = 0, M_Z = 0$$

$S_H$

$\Phi$

$$T_2 \Phi_0 \neq 0$$

$$\psi \Phi_0 \neq 0$$



$$Q \bar{\Phi}_0 = 0$$

Lost  $w, z$  mass connection!

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$$M_w \neq 0, \quad M_z = 0$$

$$M_w = g v_T$$

Forgot

- $$y_d \bar{d}_L^0 d_R^0 (u+h) + h.c.$$

$$\downarrow \text{Yew}$$

$$\bar{d}_L^{0i} y_d^{ij} d_R^{j0} (u+h)$$

$$\Downarrow$$

$$\underline{\underline{M_d}} = \underline{\underline{y_d}} u$$

$$\Downarrow$$

diagonalize

$d^0 \neq$   
physical

$$\underline{\underline{M_d}} = U_L \tilde{m}_d U_R^\dagger$$

$$H: U_L = U_R \quad \Downarrow$$

diagonal  
matrix

$$Y_d = U_L Y_d U_R^\dagger$$

"diagonal"



$$h \frac{m_d}{x} \bar{d} d$$

correct in  
physical basis

$$d_L = U_L d_L^0, \quad d_R = U_R d_R^0$$

$$\cancel{h \bar{d} s} \quad \text{forbidden}$$

predicted by SM !!!



$$\tilde{M}_d \approx \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

diagonal down mass  
matrix

- $M_t = 2M_W$

$$\Rightarrow \theta_{tu} \approx 10^{-3}$$



how to know only one  
Higgs boson?