BBSM Neutrino Course

Lecture $\overline{\underline{XIV}}$

LMU

Spring 2020

It's neutrino, stupid!

Towards a theory of neutrino mass

↓

SM perspective

$\not{P}$ violation

$\begin{pmatrix} u \\ d \end{pmatrix}_L$	u_R, d_R
$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$	e_R, $\cancel{\nu_R}$

$$\boxed{SU(2)_L \times U(1)}$$

electro-weak theory

$$\boxed{Q = T_3 + \frac{Y}{2}}$$

$10^{-5}\,GeV^{-2}$

$G_F \, J_W \, \bar{J}^W$ — effective theory

$\boxed{\text{Fermi}}$

'50s $\boxed{QED}$

$e J^\mu_{em} A_\mu \quad \frac{e^2}{4\pi} = \alpha_{em}$

messenger

$\boxed{\text{weak int} = \text{messenger}}$

Glashow '1961

$$g \, W_\mu \, J^\mu_W \to \frac{g^2}{4\pi} = \alpha_W$$

Minimal gauge theory of weak int

SO(2) = 3 gen.

↓ can it work?

$$D_\mu = \partial_\mu - i g T_a A_\mu^a$$

$$T_a \quad a = 1,2,3$$

$$\boxed{\frac{A_1 \mp i A_2}{\sqrt{2}} = W^\pm}$$

$$T_3 \longleftrightarrow A_3 \overset{?}{=} A$$ photon

why A is not photon?

(i) $W^\pm$ (A_1, A_2) — pure LH

$$\Updownarrow$$

$A_3 \rightarrow$ LH fermions

$$\boxed{A \Rightarrow L + R}$$

(ii) $\boxed{T_3 = \frac{\sigma_3}{2} \quad \text{quantized}}$

$$\Downarrow$$

$Q_{em} = T_3 =$ quantized

$$\Downarrow$$

$q = n\,{}^1\!/_2$

$q_u = {}^2\!/_3$, $q_d = -{}^1\!/_3$

$q_\nu = 0$, $q_e = -1$

$\Downarrow$

$SU(2) \times U(1)$

$[Y, T_a] = 0$

$$Y = 2(Q - T_3)$$

$= \boxed{\text{sacred}}$

$\boxed{\text{You cannot break it!}}$

$\cancel{m_d \bar{d}_L d_R}$

$\hookrightarrow$ $\underline{\text{no}}$ $d_L \longrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_L \equiv q_L$

$d = 3 \;\Big|\; \boxed{y_d \left(\bar{u} \; \bar{d} \right)_L \Phi \, d_R}$ breaks $SU(2)$

mass

$\hookrightarrow d = 4$

scalar field $\begin{cases} \cdot \text{ dim. arg} \\ \cdot \text{ Lorentz} \end{cases}$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \to U \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad U^{+} U = 1$$

- $\Phi \longrightarrow U\Phi$ Weinberg '67

  Higgs doublet

- $Y_\Phi = ?$

$$Q = T_3 + \frac{Y}{2} \Rightarrow d_R: -1/3 = \frac{Y_2}{2}$$

$$\Rightarrow Y_{d_R} = -2/3$$

$$d_L: -1/3 = -1/2 + Y/2 \Rightarrow Y_{d_L} = 1/3$$

$$(\bar{u}\ \bar{d})_L \quad \Phi \quad d_R$$

$$Y: \quad -1/3 \quad \textcircled{+1} \quad -2/3$$

$$Q\Phi = \left[\begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} + \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}\right]\Phi$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\Phi$$

$$\Downarrow$$

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$ — neutral (Qem)

$$\Downarrow \qquad T_3 \varphi_0 = -\tfrac{1}{2} \varphi_0$$

$$Y_d \left(\bar{d}_L \varphi_0 d_R + \bar{d}_R \varphi_0^* d_L \right)$$

$\Downarrow$ mass for d

$$\varphi_0 = \boxed{v} + h$$

Constant

Higgs field

$$\Phi \longrightarrow U \Phi \qquad \Phi_0 = \underbrace{\begin{pmatrix} 0 \\ v \end{pmatrix}}$$

ground state = vacuum

$$\boxed{\text{Vacuum} \neq 0}$$

$$V\Phi_0 \neq \Phi_0 \;\therefore\; T_a\Phi_0 \neq 0$$

$$\Downarrow$$

$$y_d\,(\underbrace{\bar{d}_L d_R + \bar{d}_R d_L}_{\bar{d}d})(v+h)$$

$$= y_d\,\bar{d}d\,(v+h) \Rightarrow \boxed{m_d = y_d v}$$

$$= m_d\,\bar{d}d + \boxed{\frac{m_d}{v}\,h\,\bar{d}d}$$

Static

Higgs couples to the mass of d quark

$$h \to \bar{d}d \Rightarrow \Gamma(h \to \bar{d}d) =$$

$$= \left(\frac{m_d}{v}\right)^2 \frac{m_h}{8\pi}$$

$v = ?$

u quark mass?

It's Higgs, stupid?

$$Y_u (\bar{u}_L \, \bar{d}_L) \, \epsilon \, \Phi^* u_R \quad ??$$

Y: $-\frac{1}{3}$ $\quad$ $4/3$

anti-doublet $\quad$ anti-doublet

$$\boxed{Y_\Phi = +1}$$

$$\boxed{\epsilon \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = i\sigma_2}$$

$$\Downarrow$$

$$Y_u (\bar{u}_L \, \bar{d}_L) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ h+v \end{pmatrix} u_R + h.c$$

$$= y_u (\bar{u}_L \, \bar{d}_L) \begin{pmatrix} h+v \\ 0 \end{pmatrix} u_R + h.c.$$

$$\boxed{\begin{aligned} &= g_u (\bar{u}_L u_R + \bar{u}_R d_L)(h+v) \\ &\qquad \Downarrow \\ &= m_u \bar{u} u + \frac{m_u}{v} \bar{u} u h \end{aligned}}$$

couples to mass

· <u>electron mass</u>

$$y_e (\bar{\nu} \, \bar{e})_L \, \bar{\Phi} \, e_R + h.c.$$

$$\Downarrow$$

$$m_e \bar{e} e + \frac{m_e}{v} h \bar{v} v$$

$$\boxed{v = ?}$$

$$\mathcal{L}_\Phi = \frac{1}{2}(D_\mu \Phi)^\dagger (D^\mu \Phi)$$

U(1) coupling

$$\boxed{Y_\Phi = 1} \qquad D_\mu = \partial_\mu - ig\, \underbrace{T_a\, A_\mu^a}_{SU(2)} - ig'\frac{Y}{2} B_\mu$$

SU(2) coupling

U(1)

$$T_a = \frac{\sigma_a}{2}$$

$$\Phi = \begin{pmatrix} 0 \\ v+h \end{pmatrix} \rightarrow \text{Higgs}$$

int.

$$D_\mu \Phi = \cancel{\partial_\mu \Phi} - \frac{i}{2}\begin{pmatrix} gA_3 + g'B & g(A_1 - iA_2) \\ g(A_1 + iA_2) & -gA_3 + g'B \end{pmatrix}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

int

$$\longrightarrow -\frac{i}{2}\begin{pmatrix} g(A_1 - iA_2) \\ (-gA_3 + g'B) \end{pmatrix}(v+h)$$

$$\Downarrow$$

$$\frac{1}{2}(D_\mu\Phi)^\dagger(D^\mu\Phi) \longrightarrow$$

('67 Weinberg

$$\Downarrow$$

$$\boxed{\begin{array}{c}\frac{1}{2}\,\frac{1}{4}\Big[\underset{\overset{\|}{\text{W mass}}}{g^2(A_1^2+A_2^2)} \;+\; \underset{\overset{(v+h)^2}{\uparrow}}{(gA_3-g'B)^2}\Big]\end{array}}$$

$SU(2)\times U(1)$ glashow

$$\exists A \therefore e A_\mu \bar\psi \gamma^\mu Q_{em} \psi$$

$$Q_{em} = T_3 + \frac{Y}{2}$$

$$\Downarrow$$

$$\frac{g}{\cos\theta_W} Z_\mu \bar\psi\gamma^\mu\left[T_3 - Q_{em}\sin^2\theta_W\right]\psi$$

$$\tan\theta_W = g'/g$$

$$W^{\pm} = \frac{A_1 \mp i A_2}{\sqrt{2}} \Big\} \text{physical}$$

$$\Rightarrow \boxed{\frac{1}{2}\cdot\frac{1}{4} g^2\, 2\, W^+_\mu W^{-\mu}\left(v^2 + 2vh + \dots\right)}$$

$$\Rightarrow \boxed{M_W = \frac{1}{2} g v}$$

$$\tan\theta_W = g'/g$$

$$\Rightarrow Z_\mu = \frac{(g A_3 - g' B)_\mu}{\sqrt{g^2 + g'^2}}$$

$$A_\mu \perp Z_\mu \equiv \frac{(g' A_3 + g B)_\mu}{\sqrt{g^2 + g'^2}}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} A_\mu &= \sin\theta_W\, A_{3\mu} + \cos\theta_W\, B_\mu \\ Z_\mu &\equiv \cos\theta_W\, A_{3\mu} + \sin\theta_W\, B_\mu \end{aligned}}$$

$$\Downarrow$$

$$\underbrace{M_A = 0}_{\text{photon!}} \qquad M_Z^2 = (g^2 + g'^2)\,v^2/4$$

$$\Downarrow$$

$$\boxed{M_Z \cos\theta_W = M_W}$$

bottom line

Φ = Higgs doublet

$$\Downarrow \quad \Phi_0 = \begin{pmatrix} 0 \\ \textcircled{v} \end{pmatrix}$$

arbitrary

$$\boxed{M_W = \frac{g}{2} v}$$

$$\Downarrow$$

$$\boxed{e = g \sin\theta_W}$$

$$\boxed{\frac{m_f}{v} h \bar{f} f = g \frac{m_f}{M_W} \bar{f} f}$$

theory of mass

$$\boxed{m_f \Longrightarrow \Gamma(h \to f\bar{f}) \propto m_f^2 !}$$

$$\boxed{\left(g M_W W_\mu^+ W^{\mu -} + \frac{g}{\cos\theta_W} M_Z Z^\mu Z_\mu\right) h}$$

show this!

$\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}$ "crazy" vacuum

$$\left.\begin{array}{l} \cdot\ T_a \Phi_0 \neq 0 \\ \cdot\ Y \Phi_0 \neq 0 \end{array}\right\} \quad Q_{em} = T_3 + Y/2$$

$$\Downarrow$$

$$\boxed{Q_{em} \Phi_0 = 0}$$

worked ⟺ ϕ minimal

Predictions

(i) ∃ massless photon

(ii) massive Z boson $M_Z = \dfrac{M_W}{\cos\theta_W}$

$\theta_W \simeq 30^\circ$ $M_W = 80\,\text{GeV}$, $M_Z = 90\,\text{GeV}$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2} = \frac{e^2}{8 M_W^2 \sin^2\theta_W}$$

(iii) fermion Yukawa $\propto \frac{m_f}{M_W}$

$$y_f = \frac{g}{2}\frac{m_f}{M_W}$$

$y_f^2 \le 4\pi$ $\qquad$ $\alpha_y = \frac{y^2}{4\pi} \ll 1$

perturbative

$$\Rightarrow \quad m_f^2 / M_W^2 \ll \frac{4}{g^2} 4\pi$$

$$\boxed{m_f / M_W \ll 10}$$

"miracle" $\boxed{\text{all fermion masses } \le M_W}$

fermions live around M_W

Imagine a LR symmetric world

$$q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_R \equiv q_R$$

$SU(2) \times U(1)$ gauge weak int. theory

$$\Downarrow$$

Mass term $\Longleftrightarrow$ no wall

$$\boxed{(\bar{u}\;\bar{d})_L \; \text{\textcircled{M}} \; \begin{pmatrix} u \\ d \end{pmatrix}_R = \text{inv.}}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_{L,R} \longrightarrow U \begin{pmatrix} u \\ d \end{pmatrix}_{L,R}$$

$$y_{q_L} = y_{q_R} = \frac{1}{3}$$

- $m_u = m_d$ $\underline{\underline{\text{wrong!}}}$
- why $m_f \simeq M_W$? little "miracle"

miracles happen

- split M_u and M_d

$$\Downarrow$$

$$\text{\textcircled{Higgs}}$$

$\bar{q}_L \quad T \quad e_R$ real

D

↳ D (doublet)

$D \times D = S + T$

singlet triplet

$T =$ triplet ≡ Adjoint repr.

$$\boxed{T \rightarrow U T U^{+}}$$

$$T^{+} \longrightarrow U T^{+} U^{+}$$

$\Rightarrow T = T^{+}$ irreducible

$$\mathrm{Tr}\, T \Rightarrow \mathrm{Tr}\, U T U^{+} = \mathrm{Tr}\, U^{+} U T$$

$$= \mathrm{Tr}\, T$$

$$\Rightarrow \boxed{\mathrm{Tr}\, T = 0}$$

$$T = T_a \varphi_a \qquad a = 1,2,3$$

$\uparrow$ triplet (vector)

$$\Downarrow \qquad y_{q_L} = y_{q_R} = 1/2$$

$$\mathcal{L}_y = \bar{q}_L \left(M + Y_T T \right) q_R + h.c.$$

$$\boxed{\text{need } T_0 \neq 0}$$

Higgs

$$\mathrm{Tr}\, T = 0$$

$$\Rightarrow \mathrm{Tr}\, T_0 = 0$$

$$\Rightarrow T_0 = v \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = v \sigma_3$$

$$= 2v T_3$$

$$\Downarrow$$

$$(\bar{u}\,\bar{d})_L \left(\underline{M} + y \begin{pmatrix} v_T & 0 \\ 0 & -v_T \end{pmatrix} \right) \begin{pmatrix} u \\ d \end{pmatrix}_R + h.c.$$

$$\Downarrow$$

$$\boxed{\begin{aligned} \underline{M}_u &= \underline{M} + y\, v_T \\ \underline{M}_d &= \underline{M} - y\, v_T \end{aligned}}$$

$$T = (v_T + h_T) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$T =$ SU(2) adjoint

$\Rightarrow$ $y(T) = 0$ hypercharge

$$\Downarrow$$

$$Q = \hat{T}_3 + Y/2 \qquad \hat{T}_3 = \text{generic } SU(2)\ T_3$$

$$\boxed{Q_{em}\,\mathbb{T} = \hat{T}_3\,\mathbb{T}}$$

$$Q_{em} T_0 = \hat{T}_3 T_0 = 0$$

$$T \longrightarrow U T U^{+} \qquad U = e^{i T_a \theta_a}$$

$$= e^{i \sigma_a/2\, \theta_a}$$

$$= \left(1 + i \frac{\sigma_a}{2}\theta_a + \cdots\right) T \left(1 - i \frac{\sigma_a}{2}\theta_a + \cdots\right)$$

$$= T + i\theta_a \left[\frac{\sigma_a}{2}, T\right] + \cdots$$

$$\Rightarrow \boxed{\hat{T}_a\, \mathbb{T}_{field} = \left[\frac{\sigma_a}{2}, \mathbb{T}_{field}\right]}$$

$$T_0 = v\,\sigma_3$$

$$\Rightarrow \hat{T}_3 T_0 = \left[\frac{\sigma_3}{2}\ \sigma_3\right] v = 0$$

$$\Downarrow$$

both T_3 and Y are unbroken

$$\Downarrow$$

two "photon" =

= 2 massless gauge boson

$$\Downarrow$$

$$\boxed{M_A = 0,\ M_Z = 0}$$

SM

Φ $\quad T_z \Phi_0 \neq 0$

$Y \Phi_0 \neq 0$

$$\Downarrow$$

$$\boxed{Q \Phi_0 = 0}$$

Lost W, Z mass connection!

$$\boxed{M_W \neq 0 \quad , \quad M_Z = 0}$$

$$\boxed{M_W = g v_T}$$

Forgot

- $y_d \bar{d}_L^0 d_R^0 (v+h) + h.c.$

$$\Big\downarrow \ \boxed{\text{gen}}$$

$$\bar{d}_L^{0i} \, y_d^{ij} \, d_R^{j0} (v+h)$$

$$\Downarrow$$

$$\boxed{\underline{\underline{M}}_d = \underline{\underline{y}}_d \, v}$$

$$\Downarrow$$

diagonalize

$\boxed{d^0 \neq \text{physical}}$

$$\boxed{\underline{M}_d = U_L \, \tilde{m}_d \, V_R^+}$$

diagonal matrix

H: $V_L = U_R$ $\Downarrow$

$$Y_d = U_L \underset{\text{diagonal}}{\underset{\parallel}{y_d}} U_R^\dagger$$

$$\Downarrow$$

$$\boxed{h \, \frac{m_d}{v} \, \bar{d} d}$$

correct in physical basis

$$d_L = U_L \, d_L^0, \quad d_R = U_R \, d_R^0$$

$$\boxed{\cancel{h \, \bar{d} s} \quad \text{forbidden}}$$

predicted by SM !!!

$$\tilde{M}_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

$\shortparallel$

diagonal down mass matrix

- $m_t = 2M_W$

$\Rightarrow \theta_{tu} \simeq 10^{-3}$

$\Downarrow$

how to know only one Higgs boson?