BBSM Neutrino Course

Lecture XI

LMO
Spring 2020

It's neutrino, stupid!

$\boxed{\text{Seesaw}}$

$e_L \longleftrightarrow e_L$

$\nu_L \longleftrightarrow \nu_R$ "ghost"

$T_3 = Y = 0$

$SU(2) \times U(1)$

$a = 1, 2, 3 \quad T_a \qquad Y = 2(Q - T_3)$

$[Y, T_a] = 0$

$N_L \equiv C \bar{\nu}_R^T$

$\Downarrow$

$$N_L^{Tj} M_D^{ji} C \nu_L^i + \frac{1}{2} N_L^{Ti} C M_N^{ij} N_L^j + h.c.$$

↳ Dirac mass matrix

$$M_N^T = M_N$$

$$\Downarrow$$

$$\begin{array}{c c} \nu & \\ N & \end{array} \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix} \equiv M_{\nu N}$$

allowed by $SU(2) \times U(1)$

↳ $\boxed{M_N \gg M_D}$

$$N^T C M_D \nu = \nu^T C M_D^T N$$

(linear in M_D)

$$U^T \underline{M}_{\nu N} U = D_{\nu N} \simeq \begin{pmatrix} \underline{M}_\nu & 0 \\ 0 & \underline{M}_{N} \end{pmatrix}$$

$$\boxed{\begin{aligned} &(U^{+} H U = D, \quad H = H^{+}) \\ &(U^T S U = D, \quad S^T = S) \end{aligned}}$$

$$U = \begin{pmatrix} 1 & \theta^{+} \\ -\theta & 1 \end{pmatrix} \qquad U^{+} = \begin{pmatrix} 1 & -\theta^{+} \\ \theta & 1 \end{pmatrix}$$

$$\left(\quad U U^{+} = \begin{pmatrix} 1 + \cancel{\theta^{+}\theta} & 0 \\ 0 & 1 + \cancel{\theta\theta^{+}} \end{pmatrix} \right.$$

$$U^T = \begin{pmatrix} 1 & -\theta^T \\ \theta^* & 1 \end{pmatrix}$$

fermions $m\bar{\Psi}\Psi = m\bar{\Psi}_L\Psi_R$ + h.c.

m is complex

Convention: $\boxed{m \in \mathbb{R}, \text{ positive}}$

- $H^\dagger = H \Rightarrow \boxed{H = UDU^\dagger \in \mathbb{C}}$

$$\underbrace{\begin{pmatrix} 1 & -\theta^T \\ \theta^* & 1 \end{pmatrix}}_{U^T} \underbrace{\begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}}_{M_{\nu N}} \underbrace{\begin{pmatrix} 1 & \theta^\dagger \\ -\theta & 1 \end{pmatrix}}_{U}$$

$$\simeq \begin{pmatrix} -\theta^T M_D & M_D^T - \theta^T M_N \\ M_D & M_N \end{pmatrix} \begin{pmatrix} 1 & \theta^T \\ -\theta & 1 \end{pmatrix}$$

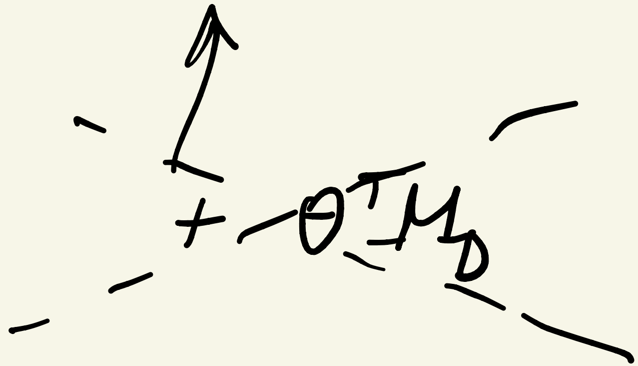
$$\simeq \left(\begin{array}{c:c} -\theta^T M_D - \theta\,\overset{\theta}{\overbrace{(M_D^T - \theta^T M_N)}} & M_D^T - \theta^T M_N \\ \hdashline M_D - M_N\theta & M_N \end{array}\right)$$

$$\Downarrow \qquad \theta\theta \simeq 0, \quad \theta M_D \theta \simeq 0$$

$$\Downarrow$$

$$\boxed{\theta = \frac{1}{M_N} M_D} \Rightarrow M_D^T = \theta^T M_N$$

$$\simeq \begin{pmatrix} -\theta^T M_D & 0 \\ 0 & M_N \end{pmatrix}$$



$$U^T M_{\nu N} U = \begin{pmatrix} M_\nu & 0 \\ 0 & M_N \end{pmatrix}$$

$$M_\nu = -\theta^T M_D$$

$$\Downarrow$$

$$\boxed{M_\nu \simeq - M_D^T \frac{1}{M_N} M_D}$$

$$\boxed{M_N \gg M_D}$$

$$U = \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix}$$

$$\begin{pmatrix} \nu \\ N \end{pmatrix} \longrightarrow U \begin{pmatrix} \nu \\ N \end{pmatrix} = \begin{pmatrix} \nu' \\ N' \end{pmatrix}$$

$$\bar{\nu}\, e\, W^+ \longrightarrow \bar{\nu} e W^+ +$$

$$\boxed{+ \bar{N} \Theta e^- W^+}$$

connection of N to real world

(ii) if I can produce N $\Rightarrow$ M_N

$$\theta \simeq \frac{1}{M_N} M_D$$

$$M_\nu = -M_D^T \frac{1}{M_N} M_D$$

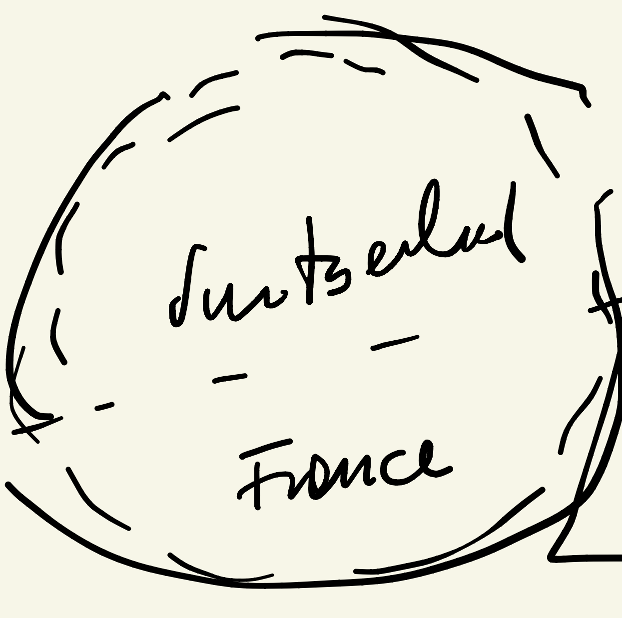
$$|\theta|^2 \simeq O\left(\frac{1}{M_N} M_D\right)^2 \simeq O\left(\frac{M_\nu}{M_N}\right)$$

$$\boxed{M_N \gtrsim M_W}$$

$$\boxed{\sigma(N) \propto |\theta|^2 \propto M_\nu / M_N \simeq 0}$$

$\boxed{\text{LHC}}$

$p + p$



27 km

$\boxed{E_{CM} \simeq 14\,\text{TeV}}$

energy loss ↗ LEP tunnel

$\sim \frac{1}{m^4}$

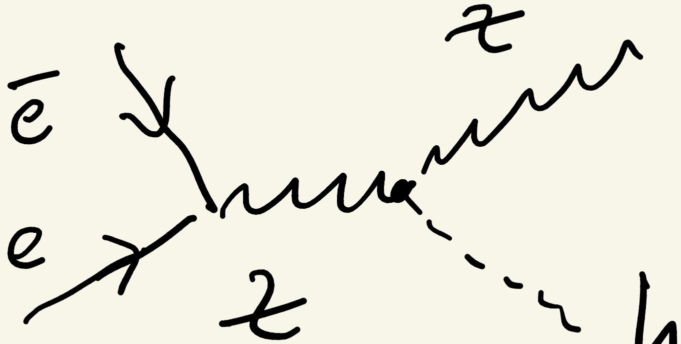
$e\bar{e}$ $\quad E_{CM} \simeq 205$ GeV

$$\boxed{\frac{E_{loss}\ LEP}{E_{loss}\ LHC} \simeq 10^{12}}$$

h = Higgs $\qquad g M_Z h Z_\mu Z^\mu$

$g \frac{m_f}{M_W} h \bar{f} f \qquad e: 10^{-5}$

$m_Z = 90$ GeV



$m_h \simeq 125$ GeV

LEP = W, Z factory (10^9)

$$M_Z = (90 \pm \leq 1)\ \text{GeV}$$

$$\boxed{\begin{array}{l}\text{Future LEP}',\ \text{Linear} \\ \simeq 500\ \text{GeV} \\ \Rightarrow \text{Higgs factory}\end{array}}$$

<u>LHC</u> $\therefore$ $m_H \leq \text{TeV}$

Finds it!

$\boxed{\begin{array}{l} \text{SM: } m_h < \text{TeV} \\ \text{LHC: to find Higgs} \end{array}}$

Higgs

gives mass to all SM particles

W, Z, t, b, τ

Discovery : $\boxed{\text{Hadron machine}}$

E = free (per quark)

→ Clean the discovery : $\boxed{\text{lepton machine}}$

$$\boxed{E_{CM} \gtrsim M_{\text{new particle}} \; (M_N)}$$

$\boxed{\sigma \text{ – large enough}}$

coupling ~ weak (em)

$e = 1/3$ $\alpha \simeq e^2/4\pi = 1/100$

$$\boxed{N : g \frac{m_D}{m_N} = g \sqrt{\frac{m_\nu}{m_N}} \leq 10^{-6}}$$

$$m_N \gtrsim 100 \text{ GeV}$$

<u>hope</u> iff N has new "weak"

$$W' \, \bar{N} \, e$$

$$M_\nu = - M_D^T \frac{1}{M_N} M_D \quad (*)$$

colliders

measure at ν oscillations, $0\nu2\beta$, Katrin ...

$\Downarrow$??

$$M_D = f(M_\nu, M_N)$$

$$\textcircled{*} \quad M_D = i\sqrt{M_N}\, O \sqrt{M_\nu}$$

$$\Downarrow$$

$$M_\nu = i^2 \underbrace{\sqrt{M_\nu}\, O^T \cancel{\sqrt{M_N}}}_{M_D{}^T} \cancel{\frac{1}{M_N}} \times$$

$$\times \cancel{\sqrt{M_N}}\, O \sqrt{M_\nu}$$

$$\boxed{\left(\sqrt{M}\right)^2 = M} \quad (\text{def.})$$

$$\boxed{\begin{gathered} M_\nu = -\sqrt{M_\nu}\, \boxed{O^T O} \sqrt{M_\nu} \\ O^T O = 1 \end{gathered}} \quad \boxed{O \in \mathbb{C}}$$

$$\boxed{M_D = i\sqrt{M_N}\, O \sqrt{M_\nu}}$$

$$\Theta = \frac{i}{M_N}\sqrt{M_N}\, O \sqrt{M_\nu}$$

↑ ambiguity

$\boxed{\text{NO prediction}}$

- $O \in \mathbb{R} \Rightarrow O = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$

$$|O_{ij}| \leq 1$$

- $O \in \mathbb{C} \Rightarrow O = \begin{pmatrix} \cosh x & i\sinh x \\ \mp i\sinh x & \cosh x \end{pmatrix}$

$$\boxed{\theta_{\nu} \to \infty \;!!\;??}$$

Twisted logic

$$\sigma(N) \propto |\theta|^2 \Longleftarrow \theta \text{ large!}$$

θ large

but

$$\begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$$

$\boxed{\text{seesaw}}$ $\boxed{M_N \gg M_D}$

$$\boxed{\begin{gathered} M_\nu = - M_D^T \frac{1}{M_N} M_D \\ M_N \gg M_D \Leftrightarrow \theta \ll 1 \end{gathered}}$$

$$\boxed{O^T O = O O^T = 1} \quad (\text{def.})$$

<u>SM</u> $\quad \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad e_R$

$$\Downarrow$$

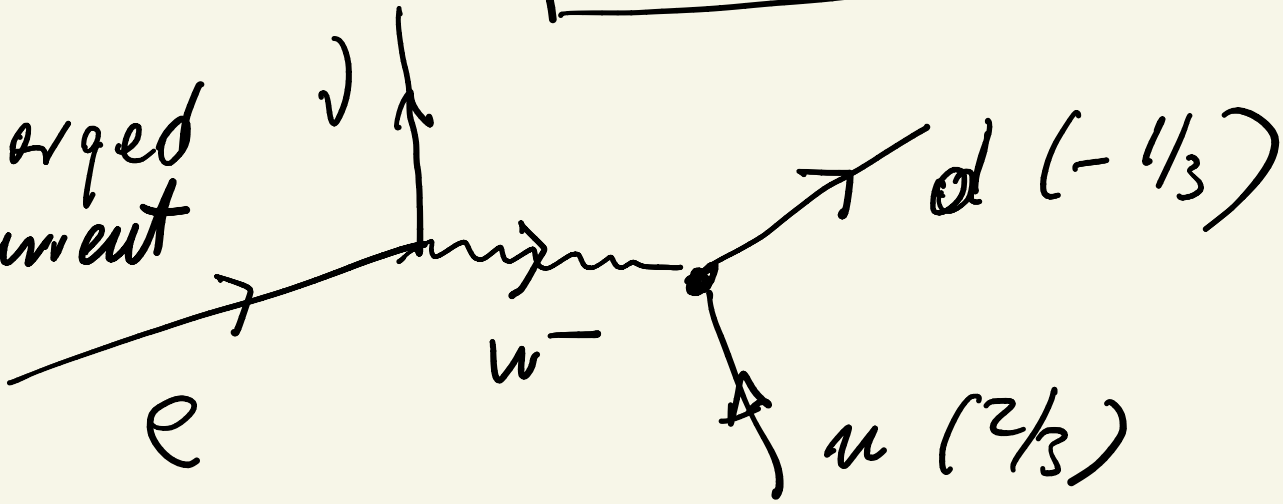
$$m_\nu = 0$$

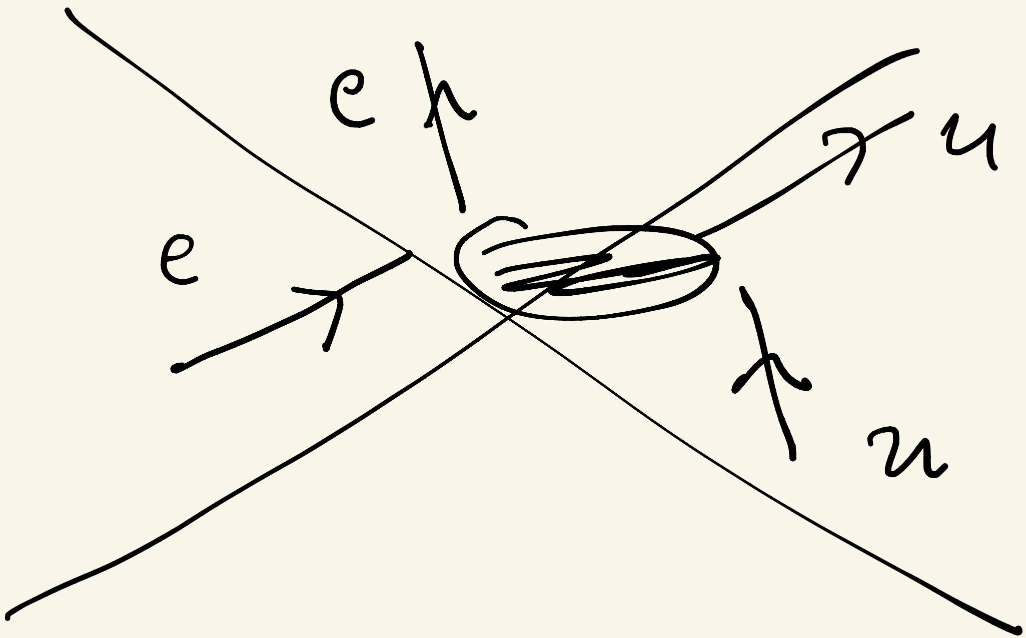
<u>exp.</u> $m_\nu \neq 0 \Longrightarrow$ what to add?

weak

$\boxed{\text{Fermi}}$

charged current





$\boxed{\text{neutral current}}$

exp [illegible]

$$g_w W^+_\mu \bar{\nu} \gamma^\mu L e$$

$g_Z \; Z_\mu \bar{e} \gamma^\mu L e$

Fermi: $\dfrac{g^2}{M_W^2}$

Given $\dfrac{g_Z^2}{M_Z^2}$ hard to fail

$\Downarrow$

$SU(2)_L$ weak symmetry

$U(1)_Y$ (em) — " —

$$D_\mu = \partial_\mu \underbrace{- i g T_a A_\mu^a}_{SU(2)} - i g' \frac{Y}{2} B_\mu$$

$$Q_{em} = T_3 + \frac{Y}{2}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$u_R,\ d_R \qquad e_R$$

$\Big\}$ weak

$$T_3 u_L = +\tfrac{1}{2} u_L \qquad T_3 e_L = -\tfrac{1}{2}$$

$$\boxed{\mathcal{L}_{SM} = i \bar{\Psi} \gamma^\mu D_\mu \Psi} \Leftarrow \text{sum}$$

$$T_a = \frac{\sigma_a}{2}$$

$T_1,\ T_2$ — off-diagonal

$T_3,\ Y/2$ — diagonal

o $\not{A}$

$$i = 1, 2$$

$$i\,(\bar{u}\ \bar{d})_L\,\gamma^\mu\left(\partial_\mu - i g\, T_i\, A_\mu^i\right)\begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$\to g\,(\bar{u}\ \bar{d})_L\,\gamma^\mu\begin{pmatrix} 0 & (A_1 - iA_2)_\mu \\ (A_1 + iA_2)_\mu & 0 \end{pmatrix}\begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$= g\,\bar{u}_L\,\gamma^\mu\,(A_1 - iA_2)\,d_L + h.c.$$

$$\equiv \frac{g}{\sqrt{2}}\,\bar{u}_L\,\gamma^\mu\,d_L\,W_\mu^+ + h.c.$$

$$W_\mu^\pm = \frac{(A_1 \mp iA_2)_\mu}{\sqrt{2}}$$

diagonal

$$\bar{f}\left[g\, T_3\, A_{3\mu} + g'\,\frac{Y}{2}\, B_\mu\right]\gamma^\mu f$$

$$\frac{Y}{2} = Q - T_3$$

$$= \bar{f}\,\gamma^\mu\Big[\underbrace{T_3\,(g\,A_3 - g'B)_\mu}_{\boxed{\text{NOT } A}} + g'\,Q\,B_\mu\Big] f$$

$\boxed{\text{Glashow '61}}$

$\boxed{\exists \text{ photon}}$

$$e A_\mu\, \bar{f}\,\gamma^\mu Q\, f$$

$$\boxed{Z_\mu = \frac{(g\,A_3 - g'B)_\mu}{\sqrt{g^2 + g'^2}}}$$

$$A_\mu \perp Z_\mu = \frac{(g' A_3 + g B)_\mu}{\sqrt{g^2 + g'^2}}$$

$$\to \bar{f} \gamma^\mu \left[\sqrt{g^2 + g'^2}\, T_3 Z_\mu + g' Q \frac{(g A - g' Z)_\mu}{\sqrt{g^2 + g'^2}} \right] f$$

$$\boxed{\tan \theta_W \equiv g'/g}$$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}; \quad \cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}$$

$$= \bar{f} \gamma^\mu \left[\left(\frac{g}{\cos\theta_W} T_3 - \sin\theta_W\, g \tan\theta_W Q \right] Z_\mu \right.$$

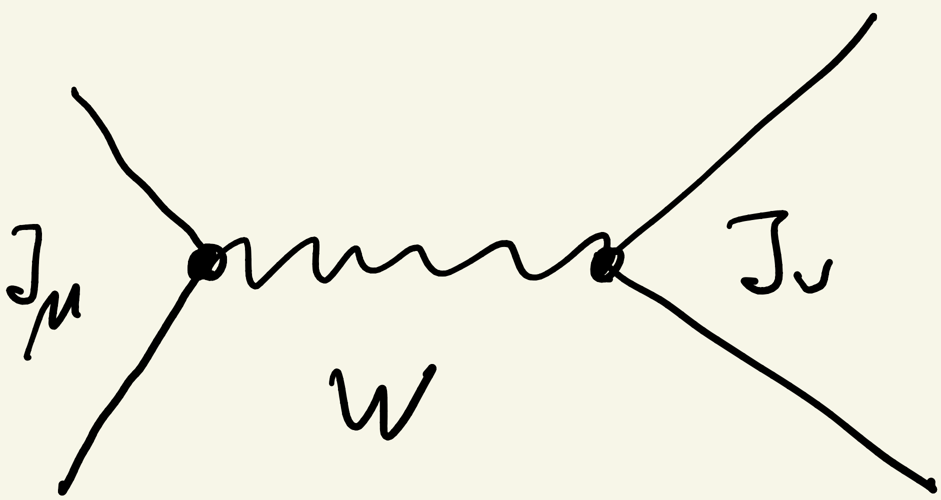
$$\left. + g \sin\theta_W Q A_\mu \right] f$$

$$= e\,\bar{f}\gamma^\mu Q f A_\mu \qquad \boxed{e = g\sin\theta_W}$$

$$+ \frac{g}{\cos\theta_W}\,\bar{f}\gamma^\mu\left(T_3 - Q\sin^2\theta_W\right) f Z_\mu$$

$\boxed{e < g}$

weak int. are strong !!!



$\frac{g}{\sqrt{2}}$

$\Downarrow$

$$\frac{g^2}{2} J^\mu_w \bar{J}^\nu_w \quad \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2}$$

$$\rightarrow \frac{g^2}{2}\frac{1}{M_W^2} J^\mu_w \bar{J}_\mu^w = \frac{4G_F}{\sqrt{2}} J^\mu_w \bar{J}_\mu^w$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{e^2}{8(M_W \sin\theta_W)^2}$$

$$\theta_W \simeq 30^\circ \Leftrightarrow \sin^2\theta_W = 0.23$$

$$\boxed{M_W = 80\ \text{GeV}}$$

$SU(2) \times U(1)$

$\Downarrow$

if W ($\leftrightarrow$ Fermi)

A ($\leftrightarrow$ em)

$\Downarrow$

new Z (neutral weak)

from given

$\Downarrow$ $\boxed{g = \text{fixed!}}$