BBSM Neutrino Course

Lecture X

26/5/2020

LMU

Spring 2020

It's neutrino, stupid!

Towards a (the?) theory of neutrino mass

Seesaw mechanism

SM **gauge theory**

$$\boxed{\ell_L \equiv \begin{pmatrix}\nu\\ e\end{pmatrix}_L} \longleftarrow \boxed{e_R} \qquad \uparrow \left(T_a = \frac{\sigma_a}{2}\right)$$

$U(1)_{em} \longrightarrow SU(2)_L$ gauge sym. of weak int.

Q_{em}

$$Q_{em} = T_3 + \frac{Y}{2} \leftarrow \text{hyper-charge} \quad (1)$$

$$[Y, T_a] = 0 \qquad \underbrace{[T_a, T_b] = i\varepsilon_{abc}T_c}_{SU(2)_L}$$

$$g$$

$$T_a l_L = \frac{\sigma_a}{2} l_L$$

$$T_a e_R = 0$$

$$\boxed{Y = 2(Q_{em} - T_3)}$$

$$\boxed{\begin{gathered} Q_e = -1, \quad Q_\nu = 0 \\ Y l_L = 2\left(0 - \tfrac{1}{2}\right) = -1 \end{gathered}}$$

$$\partial_\mu \to D_\mu = \partial_\mu \underbrace{- i g T_a A_\mu^a - i g' \frac{Y}{2} B_\mu}_{\text{SM electro-weak}}$$

$\nu_L^T C \nu_L$ — neutrino mass

Y: -1 -1

↑ forbidden by $SU(2)_L$, $U(1)_Y$

but

$\boxed{\text{Neutrino massive}}$ → $\boxed{\text{oscillations}}$

$\exists\ \nu_R \longleftrightarrow e_R$

$\underbrace{\bar{\nu}_L \nu_R + h.c.}_{\text{doublet mass term}} \Longleftrightarrow \bar{e}_L e_R + h.c. \equiv \bar{e} e$

singlet

doublet

$\boxed{\text{doublet}}$

$+ \boxed{\nu_R^T C \nu_R}$

$SU(2)_L \times U(1)_Y$

$Y \nu_R = 0$

$$\underset{\substack{\parallel \\ 0}}{Q_{em}} = \underset{\substack{\parallel \\ 0}}{T_3} + \underset{\substack{\Downarrow \\ 0}}{Y/2}$$

$\mathcal{L}_{mass}$ (neutrino) =

$$= m_D \bar{\nu}_R \nu_L + m_D^* \bar{\nu}_L \nu_R +$$

$$+ \boxed{\frac{1}{2}} M_R \, \nu_R^T C \nu_R + h.c.$$

$m_D \in C$, $M_R \in C$ in general

<u>but</u> $m_D = |m_D| e^{i\alpha}$

$$\nu_R \to e^{i\alpha} \nu_R, \quad \nu_L \to \nu_L$$

$$\Downarrow \qquad\qquad M_R \in \mathbb{C}$$

$$m_D(\overline{\nu_R}\,\nu_L + \overline{\nu_L}\,\nu_R) + \frac{1}{2}(M_R\,\nu_R^T C \nu_R + h.c.)$$

$$\nu_L \leftrightarrow \nu_R \qquad\qquad |$$

RH neutrino

$$\boxed{N_L \equiv C\overline{\nu_R}^T} \quad (2) \qquad C \equiv i\gamma_2\gamma_0$$

$$\equiv i\gamma_2\,\nu_R^*$$

LH as ν_L

$$M_R \neq m_D$$

(heavy) neutral lepton

$$\boxed{\overline{\nu_R}\,\nu_L \equiv N_L^T C\,\nu_L} \qquad \overline{\nu_R} = N_L^T C^T$$

$$C^T C = 1$$

$$M_R\nu_R^T C\nu_R + M_R^*\,\nu_R^\dagger C^\dagger \nu_R^*$$

$$/ \qquad\qquad \parallel$$

$$\underbrace{\nu_R^\dagger \gamma^0}_{} \gamma^0 C^\dagger \gamma^0 \gamma^0 \nu_R^*$$

$$\overline{\nu_R} = N_L^T C^T$$

$$\Downarrow \quad \boxed{\text{complete!}}$$

$$\boxed{N_L^T C N_L \equiv \nu_R^\dagger C^\dagger \nu_L^*}$$

$$\Downarrow$$

$$\mathcal{L}_{mass}(\nu) = m_D N_L^T C \nu_L + h.c.$$

$$+ \frac{1}{2} M_N N_L^T C N_L + h.c.$$

$$\boxed{M_N \equiv M_R^*}$$

$$= \frac{1}{2} m_D (N_L^T C \nu_L + N_L^T C \nu_L) +$$

$$\boxed{C^T = -C} \qquad + \frac{1}{2} M_N N_L^T C N_L + h.c.$$

$$= \frac{1}{2} m_D \left(N_L^T C \nu_L - \nu_L^T C^T N_L \right)$$

$$+ \frac{1}{2} M_N N_L^T C N_L + h.c.$$

$$= \frac{1}{2} \Big[m_D \left(N_L^T C \nu_L + \nu_L^T C N_L \right) + M_N N_L^T C N_L \Big]$$

$$\Downarrow$$

$$M_{\nu N} = \begin{array}{c} \nu_L \\ N_L \\ \\ \end{array} \begin{array}{cc} \left(\begin{array}{cc} 0 & m_D \\ m_D & M_N \end{array} \right) \\ \begin{array}{cc} \nu_L & \quad N_L \end{array} \end{array} \leftrightarrow \boxed{m_e}$$

$$\boxed{(i) \; M_N \gg m_D} \quad \boxed{\text{assumption}}$$

$$\begin{pmatrix}\nu\\ e\end{pmatrix}_L \curvearrowleft W \qquad m_D \simeq m_e$$

$$m_\nu \leq 10^{-6} m_e$$

$$\begin{pmatrix}\nu\\ e\end{pmatrix} \longleftrightarrow \begin{pmatrix}u\\ d\end{pmatrix}$$

$$\left.\begin{array}{l} \cdot\ m_u \simeq m_d \simeq \text{few MeV} \\ \cdot\ m_s \simeq 100\ \text{MeV},\ m_c \simeq \text{GeV} \\ \cdot\ m_b \simeq 5\ \text{GeV},\ m_t \simeq 200\ \text{GeV} \end{array}\right\}$$

SM fermions

all f $\therefore$ $m_f \leq M_W$

Q: why $m_f \leq M_W$?

A. $m_f \propto$ scale of $SU(2)_L$ breaking

$$m_f \, \underset{\underset{\text{doublet}}{\uparrow}}{\bar{f}_L} \, \underset{\underset{\text{singlet}}{\nwarrow}}{f_R} + h.c.$$

but M_N = SU(2) singlet

SU(2) singlet → large scale

$(M_N \gg m_D)$

Higgs

$$\boxed{M_{\nu N} = \begin{pmatrix} 0 & m_D \\ m_D & M_N \end{pmatrix}} \qquad T_r = M_N \qquad \det = -m_D^2$$

outsider

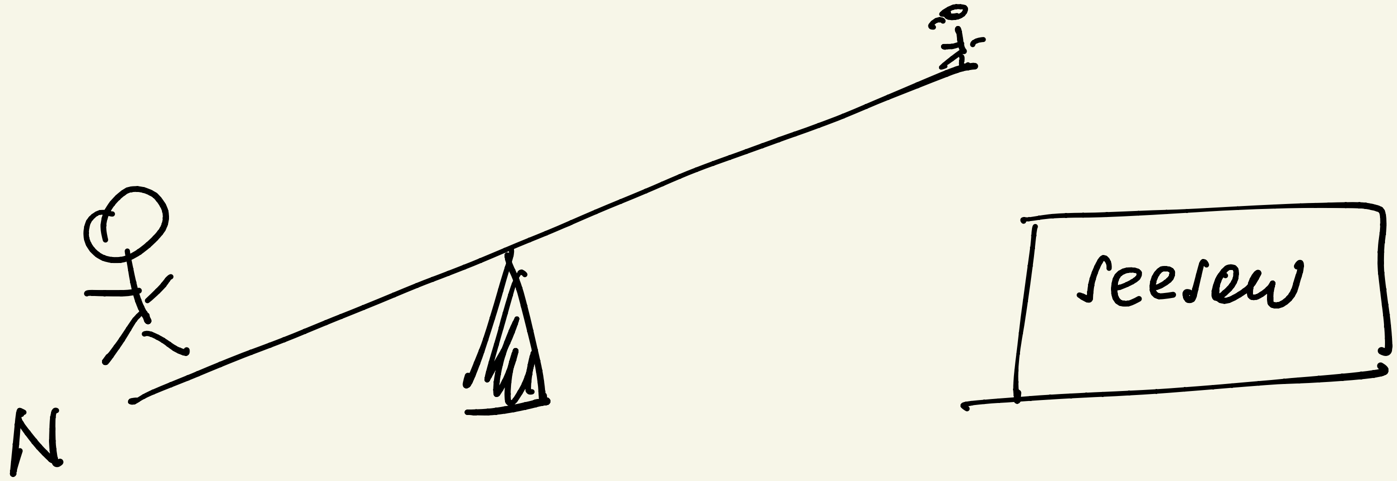
heavy $m_H \simeq M_N \cancel{+ O(m_D^2/M_N)}$

light $\boxed{m_L \simeq -m_D^2/M_N}$ $\boxed{m_L \simeq m_D}$

$$\boxed{SM \longrightarrow BSM}$$

perturbative

$$\det M_{\nu N} \simeq m_H m_L \simeq M_N(-m_D^2/M_N)$$
$$= -m_D^2 \checkmark$$



$$m_\nu \simeq -\frac{m_D^2}{\cancel{\boxed{M_N}}} \xrightarrow[M_N \to \infty]{} 0$$

- $M_N \to \infty \Rightarrow$ N decouples

  $\Rightarrow$ SM $\Rightarrow m_\nu \to 0$

- $\underbrace{\nu_L^T C \nu_L}_{\text{SU(2)}_L \text{ triplet}}$ ← neutrino mass

$T_3: \frac{1}{2} + \frac{1}{2} = 1$

m_D = doublet
$(\bar{\nu}_L \nu_R)$

Triplet ~ Doublet^2

mass : $E^2 = \vec{p}^{\,2} + \boxed{m^2}$

sign is <u>not</u> physical

- $m_e\left(\overline{e_L}\, e_R + \overline{e_R}\, e_L\right)$

$e_R \to -e_R, \quad e_L \to e_L \Rightarrow m_e \to -m_e$

- $m_e\, \overline{e_L}\, e_R + m_e^*\, \overline{e_R}\, e_L$

$$\boxed{\text{mass} > 0}$$

$\nu_L^T C \nu_L \left(- \boxed{- - -} \right)$ $\qquad \nu_L \to i\nu_L$

$\qquad\qquad\qquad\qquad\qquad\qquad \Downarrow$

$\hookrightarrow \quad - \Longrightarrow +$

$$\frac{g}{\sqrt{2}}\,\bar{e}_L \gamma^\mu \underset{\uparrow}{\nu_L}\, W^-_\mu \rightarrow \textcircled{i}\,\frac{g}{\sqrt{2}}\,\bar{e}_L \gamma^\mu \underset{\uparrow}{\nu_L}\, W^-_\mu$$

who is N ?   RH neutrino

$$\theta_{\nu N} \simeq m_D / M_N \qquad \begin{array}{c} \nu \\ N \end{array} \begin{pmatrix} 0 & \textcircled{m_D} \\ m_D & M_N \end{pmatrix}$$

$$\boxed{\theta_{\nu N} \rightarrow 0\,,\quad m_D \rightarrow 0}$$

$$\boxed{\theta_{\nu N} \rightarrow 0\,,\quad M_N \rightarrow \infty}$$

$$M_{\nu N} \begin{pmatrix} 1 \\ -\theta_{\nu N} \end{pmatrix} = m_\nu \begin{pmatrix} 1 \\ -\theta_{\nu N} \end{pmatrix} \quad (\sim \theta^2)$$

$$= -\,m_D^2 / M_N \begin{pmatrix} 1 \\ -\theta_{\nu N} \end{pmatrix} \quad (\sim \theta^2)$$

$$\| \qquad \Rightarrow \boxed{\theta_{\nu N} = m_D / M_N}$$

$$\begin{pmatrix} 0 & m_D \\ m_D & M_N \end{pmatrix} \begin{pmatrix} 1 \\ -\theta_{\nu N} \end{pmatrix} = \begin{pmatrix} -m_D \theta_{\nu N} \\ m_D - M_N \theta_{\nu N} \end{pmatrix}$$

$$\overset{\downarrow}{=} -\frac{m_D^2}{M_N} \begin{pmatrix} 1 \\ -\theta_{\nu N} \end{pmatrix}$$

$$m_D = M_N \theta_{\nu N} \quad (\text{up to } \theta^2)$$

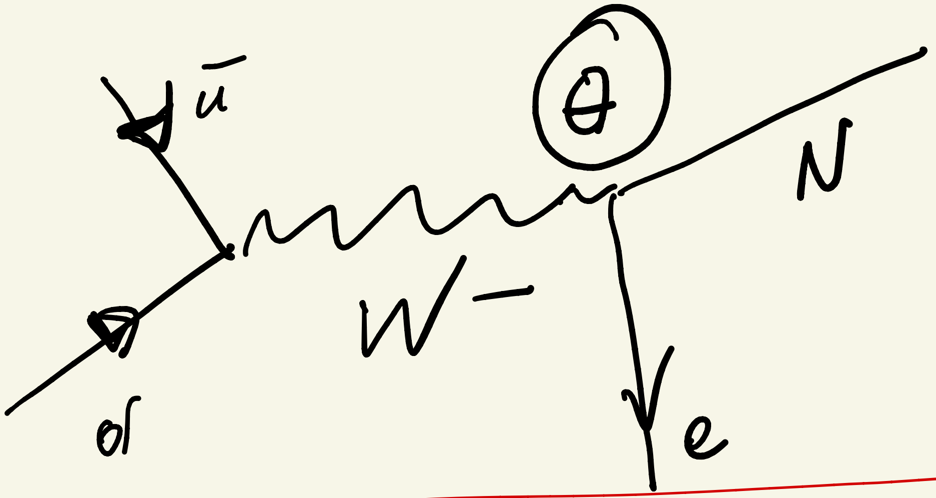
$$\boxed{\theta_{\nu N} = m_D / M_N}$$

$$\left.\begin{aligned} \nu &\to \nu - \theta N \\ N &\to N + \theta \nu \end{aligned}\right\} \quad (\theta^2 \to 0)$$

$$g\,\bar{\nu}_L \gamma^\mu e_L W_\mu^+ \longrightarrow g\,\bar{\nu}_L \gamma^\mu e_L W_\mu^+$$

$$\boxed{+\,g\,\theta_{\nu N}\,\bar{N}_L \gamma^\mu e_L W_\mu^+}$$

$$\boxed{\theta \equiv \theta_{\nu N}}$$



$(\hbar = \pi = 1)$

$$\boxed{\sigma(N) \propto |\theta^2| = \frac{m_D^2}{M_N} = \frac{m_\nu}{M_N}\;!!}$$

$$M_N \gtrsim M_W$$

$$\sigma(N) \leq \underset{\widehat{u}}{10}^{-11}\;!!$$

NO way

$$\theta_{\nu N} = \frac{m_D}{\boxed{M_N}}$$

$$N \to e + W^+$$

$$\sum_{pol} \Gamma(N \to e + W^+) \simeq \frac{g^2}{8\pi} \theta_{\nu N}^2 \frac{M_N^3}{M_W^2}$$

$$\hookrightarrow \sum \epsilon_\mu \epsilon_\nu^* = -g_{\mu\nu} + \frac{k_\mu k_\nu}{M_W^2}$$

$M_N \gg M_W$

$d[\Gamma] = \text{mass}$ $\qquad \Gamma \sim \frac{1}{t}$

$$\boxed{\Gamma_{tot}\,(N \to e W) \simeq \frac{g^2}{8\pi}\,\frac{m_\nu}{M_N}\,\frac{M_N^3}{M_W^2}}$$

measure M_N

- $m_D \lesssim M_W \quad (m_D \sim m_e)$

$\begin{pmatrix} \nu \\ e \end{pmatrix}$

$$\begin{pmatrix} m_s \simeq m_c \\ m_b \simeq m_t \end{pmatrix}$$

$$\boxed{\begin{aligned} m_\nu \simeq \frac{m_e^2}{M_N} \Rightarrow M_N &\simeq \frac{m_e^2}{m_\nu} \\ &\simeq \frac{(10^{-3}\,\mathrm{GeV})^2}{10^{-10}\,\mathrm{GeV}} \simeq 10^{4}\,\mathrm{GeV} \end{aligned}}$$

Imagine produce N

untangle seesaw $\Rightarrow$

$$|m_D| = \sqrt{m_\nu M_N}$$

More generations (n)

- $\overline{\nu_R}^{i} M_D^{ij} \nu_L^{j} \equiv \overline{\nu_R} M_D \nu_L$
- $\frac{1}{2} \nu_R^{T\, i} M_R^{ij} C \nu_R^{j} \equiv \frac{1}{2} \nu_R^T M_R \nu_R$

$i, j = 1, \dots, n$

$$N_L^i \equiv C \overline{\nu_R}^{T\, i}$$

$$\overline{\nu_R} = N_L^T C^T$$

$$\Downarrow$$

$$\rightarrow \underbrace{\overset{i}{N_L}{}^T C \overset{ij}{M_D} \overset{j}{\nu_L}}_{\|} + \frac{1}{2} \overset{i}{N_L}{}^T C \overset{ij}{M_N} \overset{j}{N_L}$$

$i, j = 1, \dots, n$

$(n = 3)$

$$\frac{1}{2}\left[N_L^T C M_D \nu_L + N_L^T C M_D \nu_L \right] + \quad + h.c.$$

$$= \frac{1}{2}\left\{ \left[N_L^T C M_D \nu_L + \nu_L^T C M_D^T N_L \right] + N_L^T C M_N N_L \right\} + h.c.$$

$$\Downarrow$$

$$\begin{array}{c} \nu \\ N \end{array} \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix} \quad (n > 1)$$

$$\equiv$$

$$M_{\nu N}$$

$$\boxed{M_{\nu N}^T = M_{\nu N}}$$

$$\boxed{\text{iff } M_N^T = M_N}$$

$$\Downarrow$$

Proof

$$N_L^{i\,T} C M_N^{ij} N_L^j =$$

$$= - N_L^{j\,T} C^T M_N^{ij} N_L^i$$

$$= + N_L^{j\,T} C \left(M_N^T\right)^{ji} N_L^i$$

$$= N_L^T C M_N^T N_L \Rightarrow \boxed{M_N^T = M_N}$$

Majorana mass matrices = symmetric

· $U M V^{\dagger} =$ Diagonal

$$UU^{\dagger} = VV^{\dagger} = 1$$

· $M = M^{\dagger} \Rightarrow V = U$

· $M = M^{T} \Rightarrow V = U^{T}$

$$\overline{f_L}\, M_D\, f_R \quad \text{(DIRAC)}$$

(MAJ)

$$f_L^{T} C M_M f_L$$

$$M_M^{T} = M_M$$

$C^{T} = -C$

$$\underline{M}_{\nu N} = \underline{M}_{\nu N}^T$$

$$U^T \underline{M}_{\nu N} U \simeq \begin{pmatrix} \underline{M}_\nu & 0 \\ 0 & \underline{M}_N \end{pmatrix}$$

$$\|$$

$$\begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix} \qquad M_N \gg M_D$$

$$U = \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix} \qquad U^+ = \begin{pmatrix} 1 & -\theta^+ \\ \theta & 1 \end{pmatrix}$$

$$UU^+ = \begin{pmatrix} 1 + \cancel{\theta^+\theta} & \cancel{\theta^+ - \theta^+} = 0 \\ \underset{\overset{\shortparallel}{0}}{\cancel{-\theta + \theta}} & 1 + \cancel{\theta\theta^+} \end{pmatrix}$$

$$\simeq \underline{1} \qquad (\theta \ll 1)$$

Show

$$\theta = \frac{1}{M_N} M_D$$ Show

$$M_\nu = - M_D^T \frac{1}{M_N} M_D$$

symmetric symmetric

$$M_\nu \propto \frac{1}{M_N}$$

$$M_\nu \propto M_D^2$$

SU(2) triplet doublet

$$M_D = f(M_\nu, M_N) \; ? \; ?$$

$\swarrow$ $\quad\quad$ $\uparrow$ $\quad$ $\uparrow$

measure one day

Θ

Andrei : conjecture

$M_D \longrightarrow$ det. up orthogonal transf.

$$M_D = i\sqrt{M_N}\,\underbrace{O}_{\text{ambiguity}}\,\sqrt{M_\nu}$$

$$\left[\begin{array}{l} O^T O = O O^T = 1 \\ O \in C \end{array}\right.$$