BBSM Neutrino Course

Lecture XIII

LMU

Spring 2020

It's neutrino, stupid!

Seesaw: from a scenario

to a genuine, predictive

theory

- $N_L = C \bar{\nu}_R^T \longleftrightarrow$ new gauge (W_R)

$$\boxed{\bullet \quad M_D = M_D^T}$$

- $M_D = M_D^\dagger$

$$\Downarrow$$

$$\boxed{\text{untangle the seesaw}}$$

$$M_\nu = - M_D^T \frac{1}{M_N} M_D \qquad (1)$$

$$\Downarrow$$

$$M_D = i \sqrt{M_N}\, O \sqrt{M_\nu} \quad (2)$$

$$O O^T = O^T O = 1$$

$$\Theta_{\nu N} = \frac{1}{M_N} M_D \qquad (3)$$

$$\boxed{\begin{array}{c} \bar{\nu} \gamma^\mu e\, W_\mu^+ \to \Theta_{\nu N} \bar{N} \gamma^\mu e\, W_\mu^+ \\ \Downarrow \\ N \to e + W^+ \quad \text{predicted} \end{array}}$$

• $M_D^T = M_D \Longrightarrow$

$$M_V = -M_D \frac{1}{M_N} M_D \Big/ M_N$$

$$\frac{1}{M_N} M_V = -\left(\frac{1}{M_N} M_D\right)\left(\frac{1}{M_N} M_D\right)$$

$$\frac{1}{M_N} M_D = i\sqrt{\frac{1}{M_N} M_V}$$

$M_D = i\sqrt{M_N}$
$\theta\sqrt{M_V}$

$$\boxed{M_D = i\, M_N \sqrt{\frac{1}{M_N} M_V}} \quad (4)$$

$\Downarrow$

$$\boxed{\theta_{VN} = i\sqrt{\frac{1}{M_N} M_V}} \quad (5)$$

$\Downarrow$

SOLVED!

$$\sqrt{M_N}\, O \,\sqrt{M_\nu} = M_N \sqrt{\frac{1}{M_N} M_\nu}$$

$$\boxed{O = \sqrt{M_N}\,\sqrt{\frac{1}{M_N} M_\nu}\ \, 1/\sqrt{M_\nu}}$$

$$O \text{ "=" } 1$$

$$\boxed{O O^T = 1} \quad (*)$$

Nemevšek, Tello, G.S /2012

$$\boxed{M_N^T = M_N, \quad M_\nu^T = M_\nu}$$

$$\psi_i^T C M_{ij} \psi_j = \text{Majorana}$$

$$C^T = -C, \quad \{\psi_i, \psi_j\} = 0$$

$$\boxed{\bullet\ M_D = M_D^{+}}$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad\qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$\overrightarrow{W_L} \qquad\qquad \overrightarrow{W_R}$$

$$L \longleftrightarrow R$$

$$\bullet\ \bar{\nu}_R M_D \nu_L + \bar{\nu}_L M_D^{+} \nu_R$$

$$\nu_L \longleftrightarrow \nu_R \qquad \text{CLR} =$$

$$\Downarrow \qquad\qquad = \text{parity})$$

$$\boxed{M_D = M_D^{+}}$$

Tello, G.S

ψ = spinor $\Rightarrow$ $\psi^c = C\bar{\psi}^T$

$$L \longleftrightarrow R$$

$$\Downarrow$$

$$\text{(i)} \quad \psi_L \underset{(\gamma^0)}{\longleftrightarrow} \psi_R \qquad \text{Parity}$$

$$\text{(ii)} \quad \psi_L \longleftrightarrow C\,\bar{\psi}_R^{\,T} \qquad C$$

$$= i\sigma_2 \psi_R^*$$

charge conjugation

$$\Downarrow$$

$$\boxed{M_D = M_D^T}$$

$$\psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix} \qquad \psi_R = \begin{pmatrix} 0 \\ u_R \end{pmatrix}$$

$$i\sigma_2 \psi_R^* = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} u \\ R^* \end{pmatrix}$$

$$= \begin{pmatrix} i\sigma_2 u_R^* \\ 0 \end{pmatrix}$$

$$\psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix}, \quad \psi_R = \begin{pmatrix} 0 \\ u_R \end{pmatrix}$$

$$\frac{1+\gamma_5}{2}\psi \qquad \frac{1-\gamma_5}{2}\psi$$

$$\boxed{u_L \longleftrightarrow u_R}$$

$$\psi_L \longleftrightarrow \gamma^0 \psi_R$$

short-hand:

$P:\ \psi_L \leftrightarrow \psi_R$

$C:\ \psi_L \leftrightarrow \psi_R^*$

Parity $\quad A_0 \longrightarrow A_0, \quad A_i \to -A_i$

$$\bar\psi \gamma^0 \psi \to \bar\psi \gamma^0 \psi, \quad \bar\psi \gamma^i \psi \to \text{-''-}$$

$$\Downarrow$$

$$\boxed{\psi \to \gamma^0 \psi \quad (P)}$$

$$\bar\psi \gamma^0 \psi = \psi^+ \psi \to \psi^+ \psi \checkmark$$

$$\bar\psi \gamma^i \psi = \psi^+ \gamma^0 \gamma^i \psi \to \psi^+ \gamma^0 \gamma^0 \gamma^i \gamma^0 \psi$$

$$= -\psi^+ \gamma^0 \gamma^i \psi = -\bar\psi \gamma^i \psi$$

$\checkmark$

$$\boxed{\begin{aligned} \psi_L \equiv L \psi \to L \gamma^0 \psi &= \\ &= \gamma^0 R \psi = \gamma^0 \psi_R \end{aligned}}$$

$$\{\gamma^0, \gamma_5\} = 0 \qquad \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix} \qquad \psi_R = \begin{pmatrix} 0 \\ u_R \end{pmatrix}$$

$$\begin{pmatrix} u_L \\ 0 \end{pmatrix} = \psi_L \to \gamma^0 \psi_R = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ u_R \end{pmatrix}$$

$$= \begin{pmatrix} u_R \\ 0 \end{pmatrix}$$

$$\boxed{u_L \leftarrow u_R} \qquad (P)$$

$$R \begin{pmatrix} u_R \\ 0 \end{pmatrix} = 0$$

$$C: \quad \psi \to \psi^c \equiv C \bar{\psi}^T$$

$$\psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix}$$

$P: \quad \psi \to \gamma^0 \psi \Rightarrow u_L \to u_R$

$$\boxed{\bar{\psi} \gamma^\mu \psi = \text{4-vector} \equiv j^\mu}$$

<u>Dirac</u>: $\boxed{\psi_D = \begin{pmatrix} u_L \\ u_R \end{pmatrix}}$ $\boxed{\text{QED}}$

$$\boxed{\underline{P} \Rightarrow \text{good} \Leftarrow C}$$

weak

$e = \begin{pmatrix} e_L \\ e_R \end{pmatrix} \longleftrightarrow$ massive particles

$e_R \to C \bar{e}_R^{\,T} \equiv e^C_L$

positron

$C: \quad \psi_L \to C\bar{\psi}_R^T$

$P: \quad \psi_L \to \psi_R$

$$\Rightarrow \quad \boxed{\psi_L \xrightarrow{CP} C\bar{\psi}_L^T}$$

$$\epsilon_{\cancel{CP}} \simeq 10^{-3}$$

Bottom line:

$P: \quad M_D = M_D^\dagger$

$C: \quad M_D = M_D^T$

solves

untangle seesaw:

$$\boxed{M_D = f(M_\nu, M_N)}$$

$$\begin{pmatrix}\nu\\ e\end{pmatrix}_R \Rightarrow \frac{g}{\sqrt{2}}\, W^+_{R\mu}\, \bar{\nu}_R\, \gamma^\mu\, e_R$$

new

Majorana

$$N = N_L + C\,\bar{N}_L^{\,T}$$

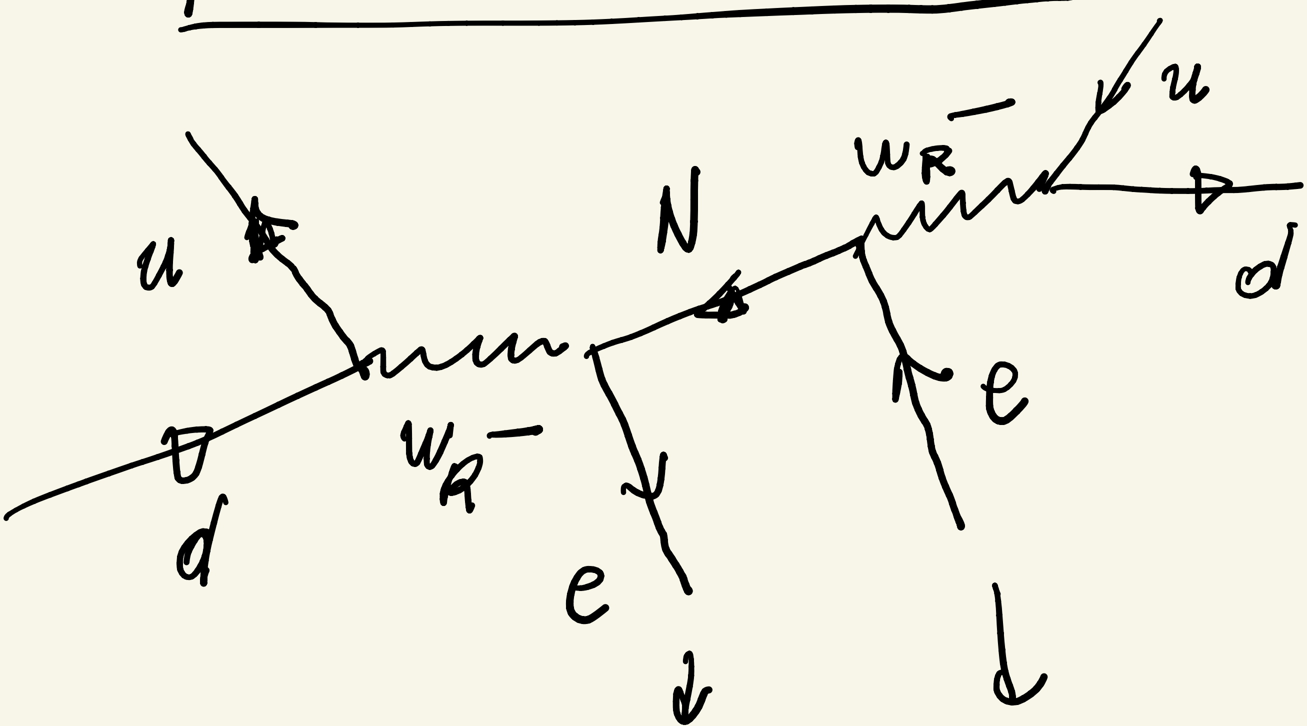
$$= C\,\bar{\nu}_R^{\,T} + \nu_R$$

$$\boxed{M_{W_R} > 4\,\mathrm{TeV}}$$

LHC

50% particle

50% anti-particle



$$e + \bar{e}$$

$$W^+_{R\mu} \bar{N} \gamma^\mu e_R + W^-_{R\mu} \bar{e}_R \gamma^\mu N_R$$

$$\| $$

$$(-1) \overline{N^c_L} \gamma^\mu e^c_L \rightarrow C \bar{e}_R^T$$

$$\boxed{N^c_L \equiv N_L} \qquad \boxed{N \equiv N^c}$$

$$W^-_{R\mu} \bar{e}_R \gamma^\mu N_R = - \overline{N_L} \gamma^\mu e^c_L W^-_{R\mu}$$

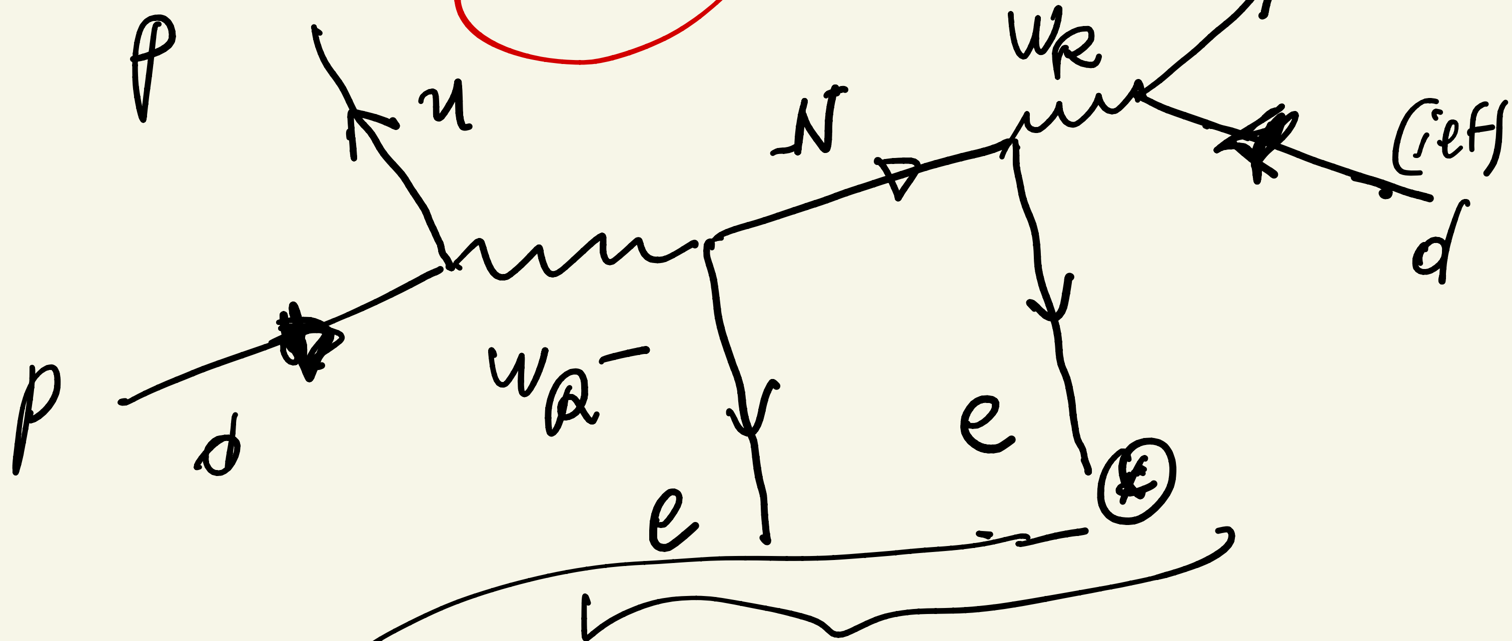
① $N \rightarrow e + W_R^+$

② $ \rightarrow e^c + W_R^-$

$$\Big\} \quad \Gamma_{①} = \Gamma_{②}$$

$\boxed{\text{KS process}}$

LHC



$e + e$

$\boxed{\text{LNV} \quad \Delta L = 2}$

$\boxed{\text{dream coming true}}$

50% e
50% e^c

$\boxed{N = \text{Majorana}}$

$p + p \to \boxed{e + e} + j + j$

(ε) (q)

jets at hadrons

$1/38$ Fury

$$\boxed{\begin{array}{l} d + d \to u + u + e + e \\ u + u \to p + p + e + e \end{array}}$$

neutrinoless double beta

$$\boxed{LHC \longleftrightarrow 0\nu 2\beta}$$

self-contained theory

$W_L \longleftrightarrow W_R$

$f_L \longleftarrow f_R$

$\Downarrow$

find N + untangle seesaw

$\Updownarrow$

probe the origin (and nature) of ν mass

$\Updownarrow$

Higgs - Weinberg origin of e mass

$$\boxed{E \gg M_{W_R} \longrightarrow \text{LR symmetric world}}$$

Higgs mechanism =

= spont. breaking of a gauge symmetry

symmetric theory under group G

$\rightarrow$ vacuum is <u>not</u> symmetric

SM $\Rightarrow$ Higgs boson

$$\Gamma(h \to p\,\bar{p}) \propto m_p^2$$

particle antiparticle

$$m_e = y v \Rightarrow \Gamma(h \to e\bar{e}) =$$

$$= \frac{1}{8\pi} g^2 \frac{m_e^2}{M_W^2} m_h$$

Probing origin of mass

$\Rightarrow$ compute a physical process by knowing the mass

$\cdot \; M_D = M_D^T \Rightarrow M_D$

$$\Downarrow$$

$$\Theta_{\nu N} = i \sqrt{\frac{1}{M_N} M_\nu}$$

$$\Rightarrow \Gamma(N \to W + e) =$$

$$= |\Theta_{\nu N}^2| M_N \; \ldots$$

- Higgs → give mass to W_L!

- new Higgs → give mass to W_R

Weinberg: give me m_e $\Rightarrow$

compute $h \to ee$

Goren: give me m_D, m_N $\Rightarrow$

compute $N \to e$ - - -

- - - - -

$$\boxed{M_D = \,? \Leftarrow \text{Higgs (SM)}}$$

<u>Majorana</u>: I believe ν can be Majorana particle

<u>LR</u>: $\Rightarrow$ ν is Majorana

New Physics (NP) $\Longleftrightarrow$ new Higgs bosons

massive A = Proca

$$\Delta^{(A)}_{\mu\nu} \propto \frac{g_{\mu\nu} - k_\mu k_\nu / M^2}{k^2 - M^2}$$

$\rightarrow$ Stuckelberg trick

$$m_A^2 \left(A_\mu - \frac{1}{m_A}\partial_\mu G\right)^2$$

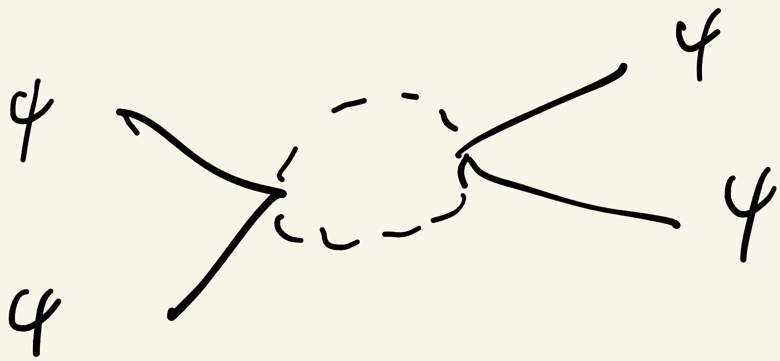
$\Rightarrow$ gauge fixing (R_ξ)

$$\Rightarrow \quad \Delta^{(A)}_{\mu\nu} \propto \frac{1}{k^2} \qquad D(G) \propto \frac{1}{k^2}$$

$$\bar{\psi}_L \psi_R \, e^{iG/v}$$

$$\parallel$$

$$\left(1 + G/v + G^2/v^2 + G^3/v^3\right)$$



$\Downarrow$

Higgs medium

G $\Rightarrow$ Higgs boson

- $\boxed{\text{High } T \quad QFT}$

Deep subject

Cosmology $\leftrightarrow$ universe was hot

- $U(1) \longleftrightarrow$ non-Abelian

$$\underbrace{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu}_{\text{Maxwell}}$$

$$\propto [D_\mu, D_\nu]$$

$$D_\mu = \partial_\mu - ieQA_\mu$$

$$[QA_\mu, QA_\nu] = 0$$

$$F_{\mu\nu}^a T^a \propto$$

$$\propto [D_\mu, D_\nu]$$

$$D_\mu = \partial_\mu - igT^a A_\mu^a$$

$$[T_a A_\mu^a, T_b A_\nu^b] = i f_{abc} A_\mu^a A_\nu^b$$

SO(2) !
$f_{abc} = \varepsilon_{abc}$

$$\Downarrow$$

$$\boxed{F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c}$$

SO(2) $\quad Q_3 = T_3$ = diagonal

(A) A_3 "photon" $\quad \boxed{A_1, A_2}$

$$W^\pm = \frac{A_1 \mp i A_2}{\sqrt{2}}$$

$$F_{\mu\nu}^3 = \partial_\mu A_\nu^3 - \partial_\nu A_\mu^3 + g \underbrace{(A_\mu^1 A_\nu^2 - A_\nu^1 A_\mu^2)}_{W_\mu^+ W_\nu^- + \leftrightarrow}$$

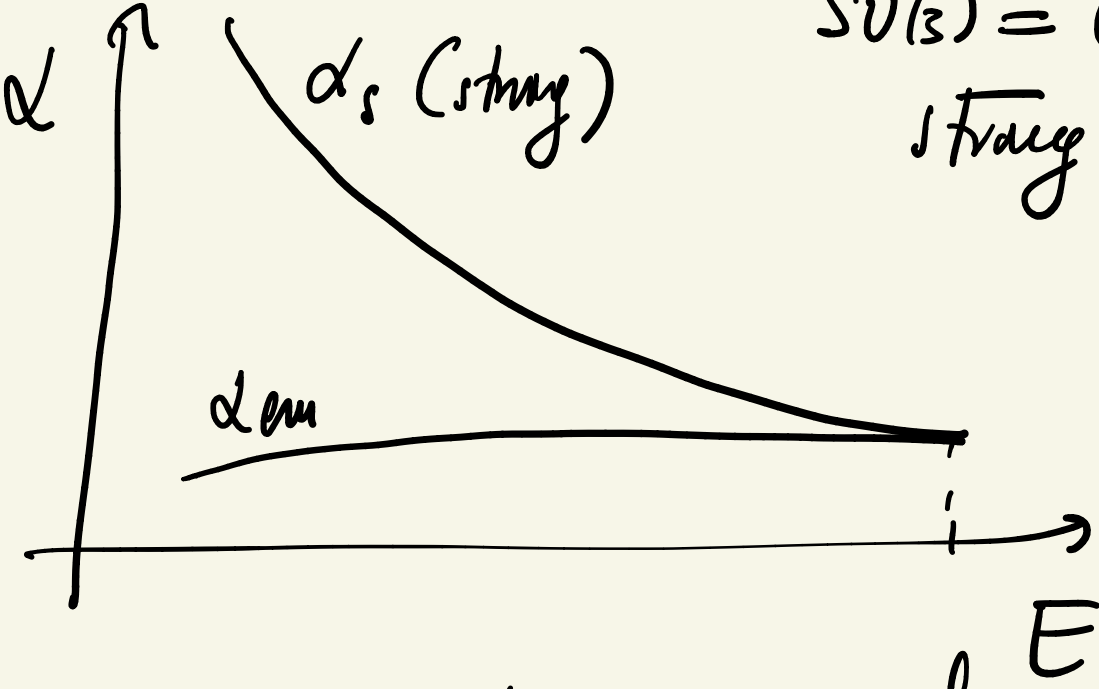
$$(F_{\mu\nu}^{3})^2 \longrightarrow \boxed{A_3 (A)\ W^+ W^-}$$

$$\frac{1}{\alpha(E_2)} - \frac{1}{\alpha(E_1)} = \frac{b}{2\pi} \ln E_2/E_1$$

$$b = \frac{11}{3}\, GB - \frac{2}{3}\, F - \frac{1}{3}\, S$$

gauge boson   fermion   scalar

$$\Downarrow$$



$SU(3) = QCD$
strong force

$E \to 0 \quad \alpha_s \to \infty$ confines

$r \to \infty \quad \alpha_{em} \to 0$ free

$\boxed{\text{color} - \text{quarks carry 3 colors}}$

$\boxed{W^- \to \bar{u} d}$ $\times 3$ colors

$$\Delta^{++} = u^\alpha u^\beta u^\gamma \epsilon_{\alpha\beta\gamma}$$

$\alpha, \beta = 1, 2, 3$
r, y, b

$u \to SU(3)$ symmetry

$q \to U q$ $\boxed{U^+ U = 1 = U U^+ \quad \det U = 1}$

$\epsilon_{\alpha\beta\gamma}\, u^\alpha u^\beta u^\gamma \to$

$\underbrace{\epsilon_{\alpha\beta\gamma}\, U_{\alpha\alpha'} U_{\beta\beta'} U_{\gamma\gamma'}}\, u^{\alpha'} u^{\beta'} u^{\gamma'}$

$\epsilon_{\alpha'\beta'\gamma'} \det U \, u^{\alpha'} u^{\beta'} u^{\gamma'}$

$= 1$

$=$ inv.

- $e\bar{e} \to$ hadrons (2×3)
- $\pi^0 \to 2\gamma$ $\pi^0 = u\bar{u} - d\bar{d}$

$\hookrightarrow \times 3$ — good rate

- $\bar{q}\, q$ mesons

  $\hookrightarrow \bar{q}\, \underbrace{U^\dagger U}_{1} q = \bar{q}\, q$

- $\underbrace{q\, q\, q}_{\text{anti-sym.}}$ baryons

- No $\cancel{q}$ $\cancel{\bar{q}}$

  $\cancel{q\, q}$ $\cancel{\bar{q}\, \bar{q}}$

$\boxed{\text{3 colors}}$ $\boxed{\text{minimal}}$

why not $SO(3) = SU(2)$?

$\underline{SO(3)}$ $\quad V = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ $\quad \vec{V}^2 = \text{inv.}$

$\vec{V}$ $\qquad V^T V$

$\cancel{q^T q}$ allowed

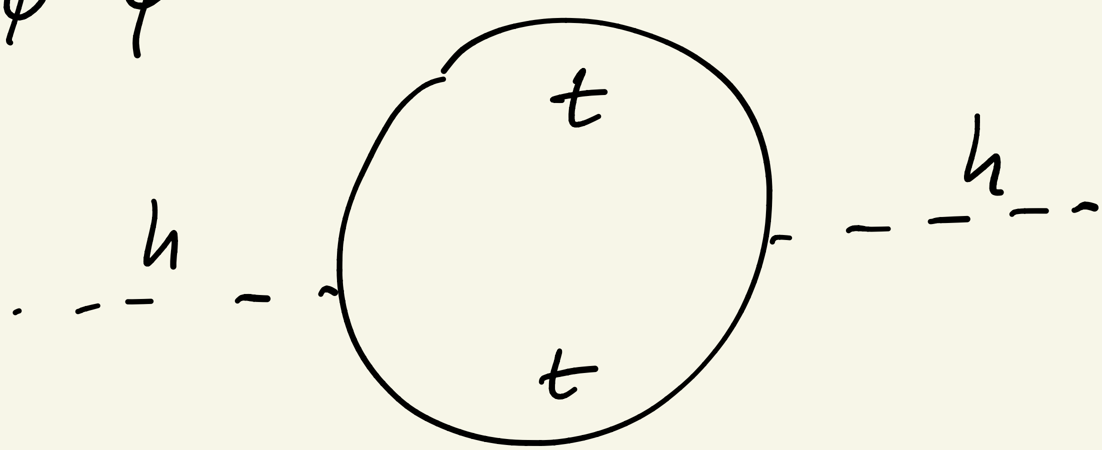
$\Downarrow$ $\boxed{\text{minimal group} = SU(3)}$

$q^T q$ $\qquad \underbrace{\bar{q}\, q \text{ mass term}}_{\text{inv. under color}}$

$\cdots X$ -

$$\boxed{\begin{aligned} m_h &= \lambda v \\ m_W &= g v \\ &\Downarrow \\ m_h &= \frac{\lambda}{g} m_W \end{aligned}}$$

$m_\phi^2 \phi^+ \phi$



$$m_\phi^2 = m_0^2 + \frac{y_t^2}{16\pi^2} \Lambda^2$$

$m_\phi^2 \sim 10^4\,\text{GeV}^2$

$\Lambda^2 \sim M_{NP}^2$

$$\boxed{\Lambda^2 = M_p, M_{GUT}}$$

$$\boxed{\Lambda = 5\ \text{TeV}}$$

$$\frac{1}{100}(5\,\text{TeV})^2 = \frac{1}{100} 10^6 \cdot 10\,\text{GeV}^2$$

$$\simeq 10^5\,\text{GeV}^2$$

SM : m_W , Φ SM

NP : M_{New} , Δ new Higgs

$$\Phi^+\Phi m_0^2 + \Phi^+\Phi \Delta^+\Delta \alpha$$

$$\Rightarrow \boxed{\begin{aligned} m_\Phi^2 &= m_0^2 + \alpha^2 \langle\Delta\rangle^2 \\ &= m_0^2 + \alpha^2 M_{new}^2 \end{aligned}}$$

$\Downarrow$ $\qquad M_{new} \parallel M_{GUT}$

Huge hierarchy

$\boxed{\Phi^+\Phi}$ inv. $\Rightarrow$ $\boxed{\Phi^+\Phi \Delta^+\Delta}$

$m_f \bar{f}_L f_R$ $\quad m_f \to 0 \Rightarrow f_R \to e^{i\alpha} f_R$

$f_L \to f_L$

$$\boxed{m_f = m_f^0 \left[1 + \frac{\alpha}{\pi} \ln \frac{\Lambda}{m_f} \right]}$$

1930 Weisskopf

SUSY

scalars $\leftrightarrow$ fermions

Hierarchy: why is $M_W \ll M_P$?

$\cdot \; m_f^0 \to 0 \Rightarrow$ chiral sym.

$\Rightarrow$ loops respect chiral sym.

$$m_f = m_f \phi^2 \left[1 + \ldots \left(\frac{\alpha}{2\pi}\right)^{27} \ln \Lambda \right]$$

$$\downarrow \qquad \Leftarrow \qquad \downarrow$$

$$0 \qquad\qquad\qquad 0$$

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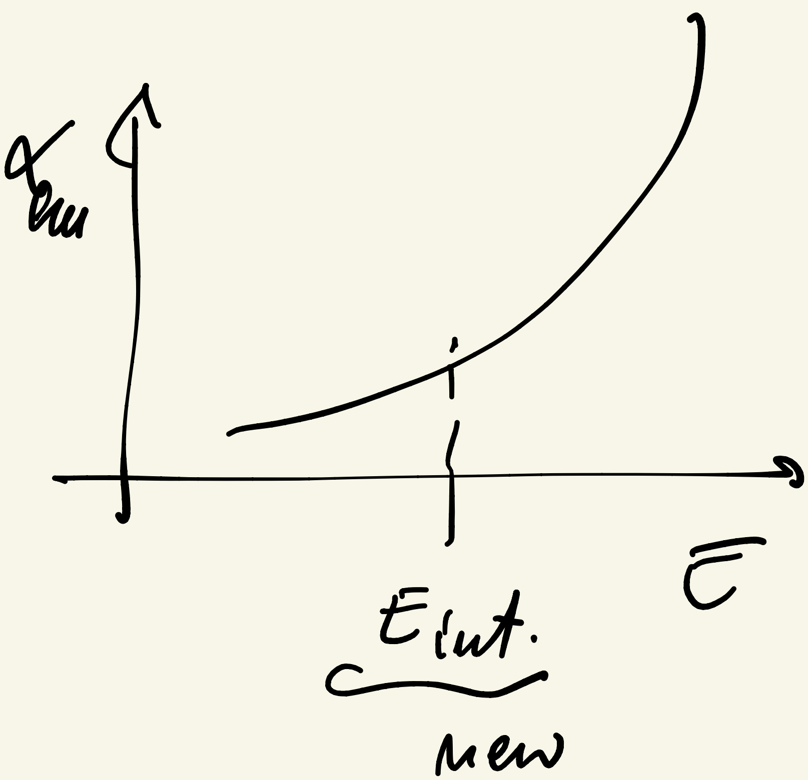
extrapolate SM $\rightarrow$ high E

$\rightarrow$ large values of ϕ



$\Lambda_L \simeq 10^{130}$ GeV

$\Downarrow$

$\alpha_{em} \rightarrow \infty$

(Landau pole)

$\downarrow$

irrelevant — there
will be new physics before