FROM UNIVERSE

TO PLANETS

LECTURE 4

REVIEW

Samples

Condensation

Lab & IDPs (interplanetary dust particles)

Meteorites

10^{-15} g

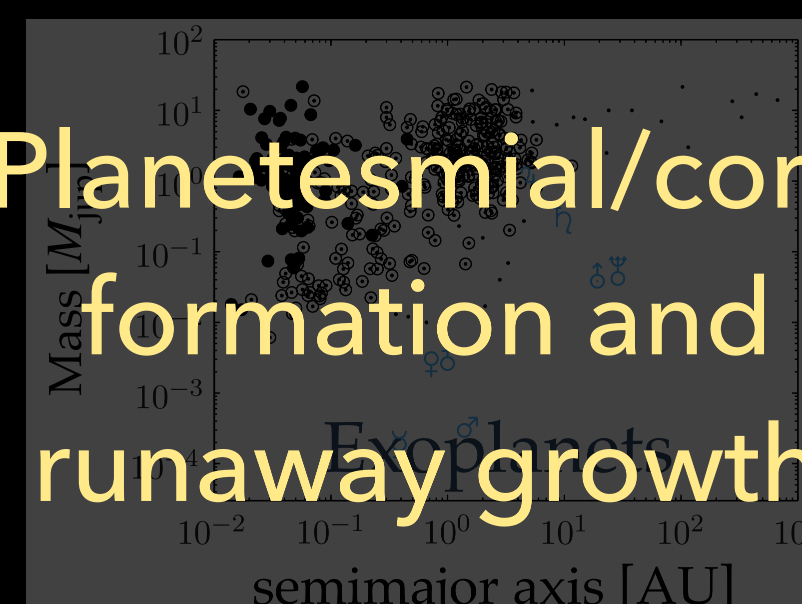
Collisional growth and fragmentation

only theory

10^{27} g

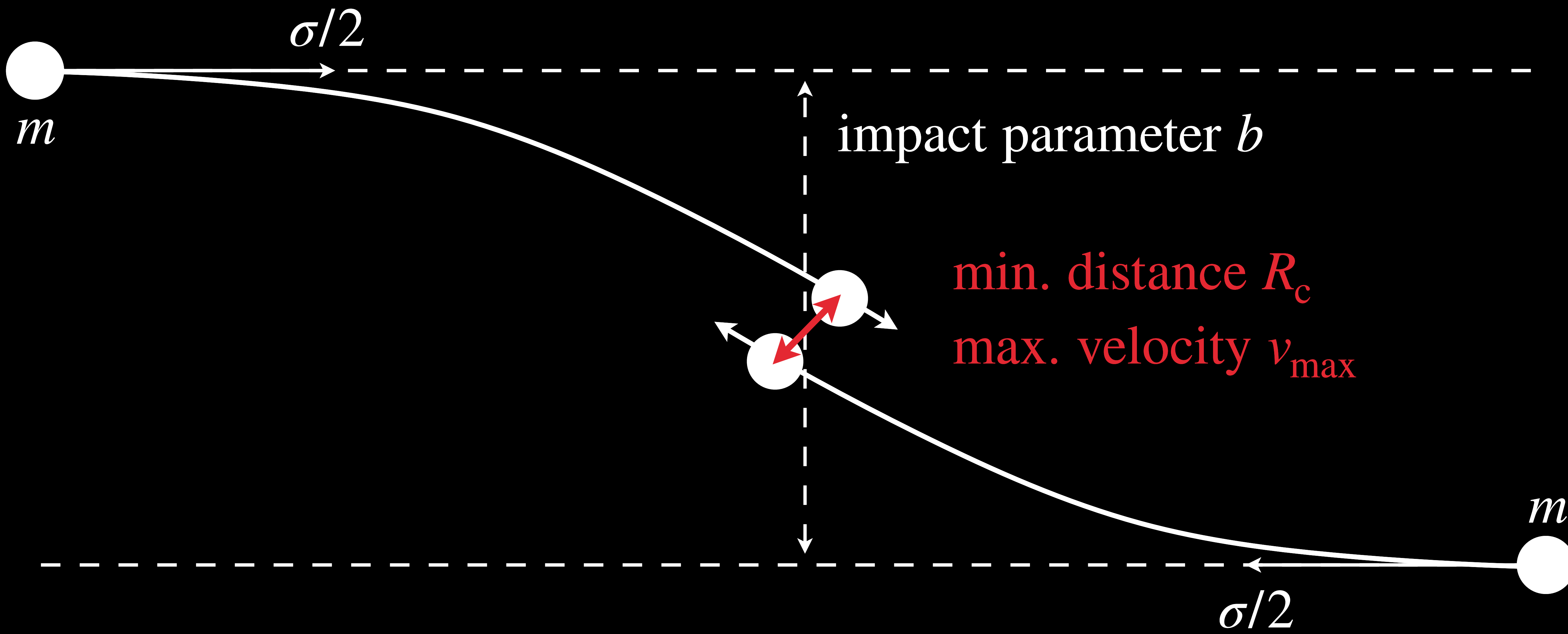
μm cm m km

Observations: IR and (sub-)mm



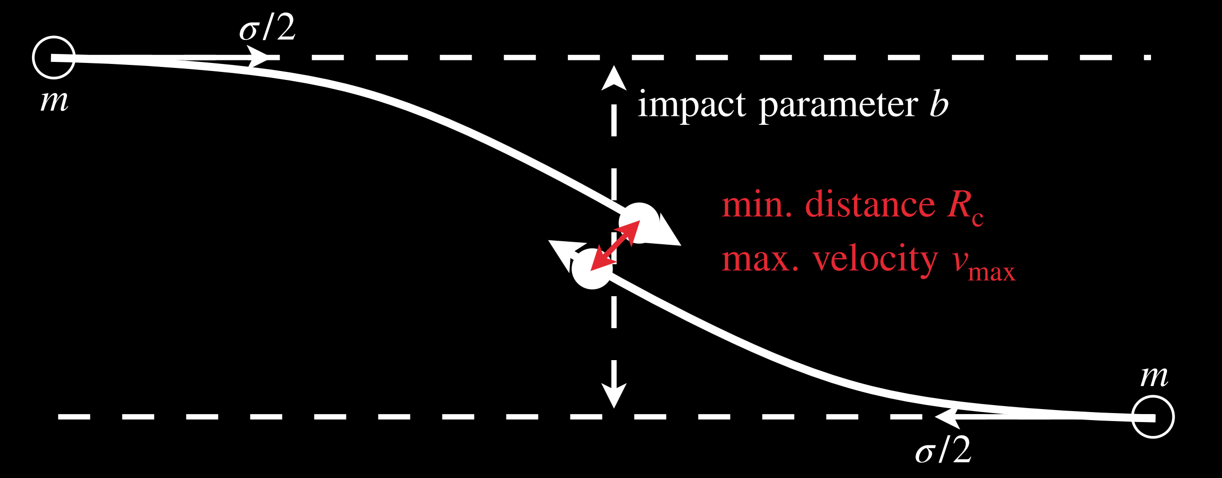
Planetesmial/core formation and runaway growth

REVIEW

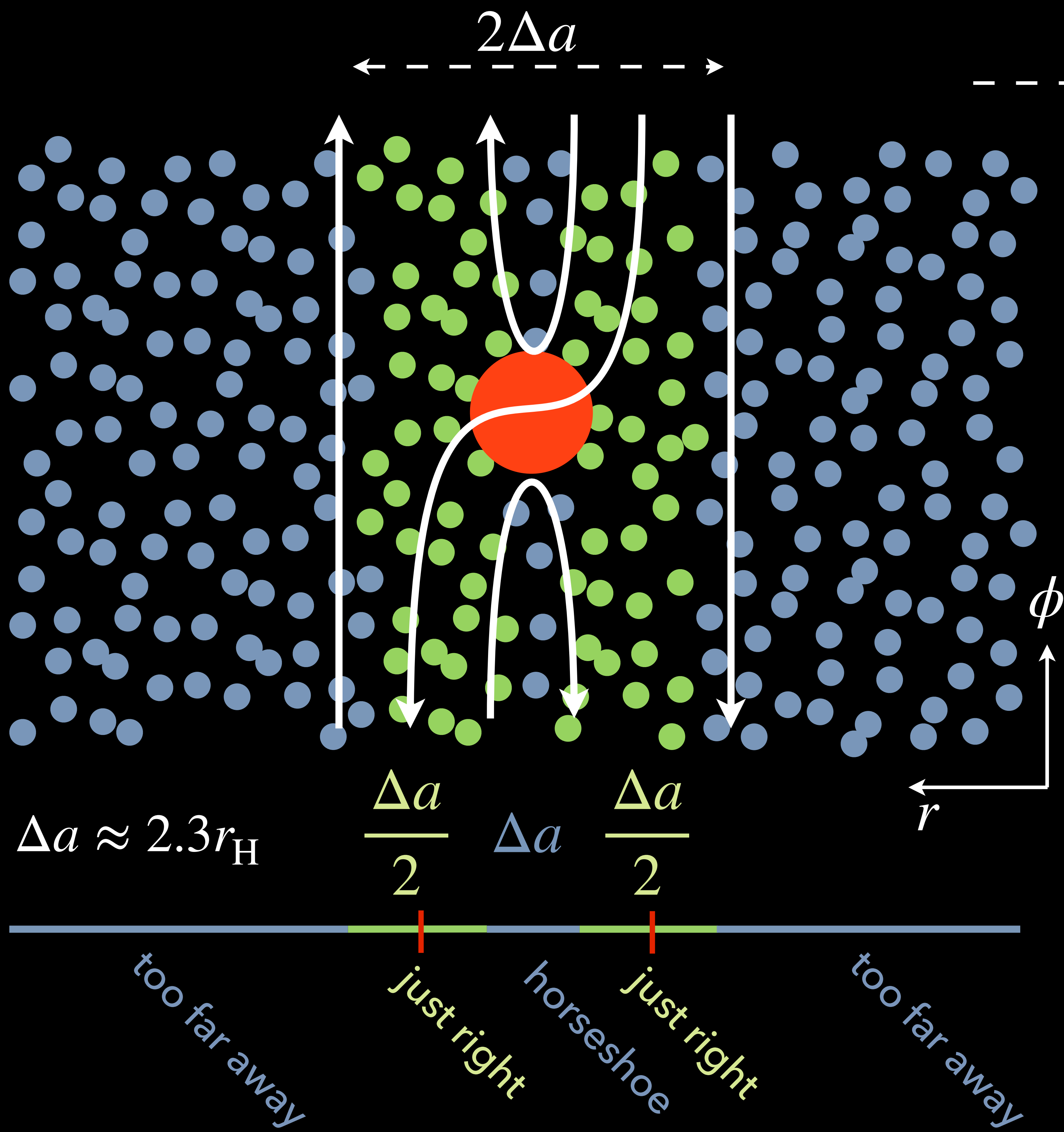


σ/2
m
impact parameter b
min. distance R_c
max. velocity v_{max}
m
σ/2

REVIEW

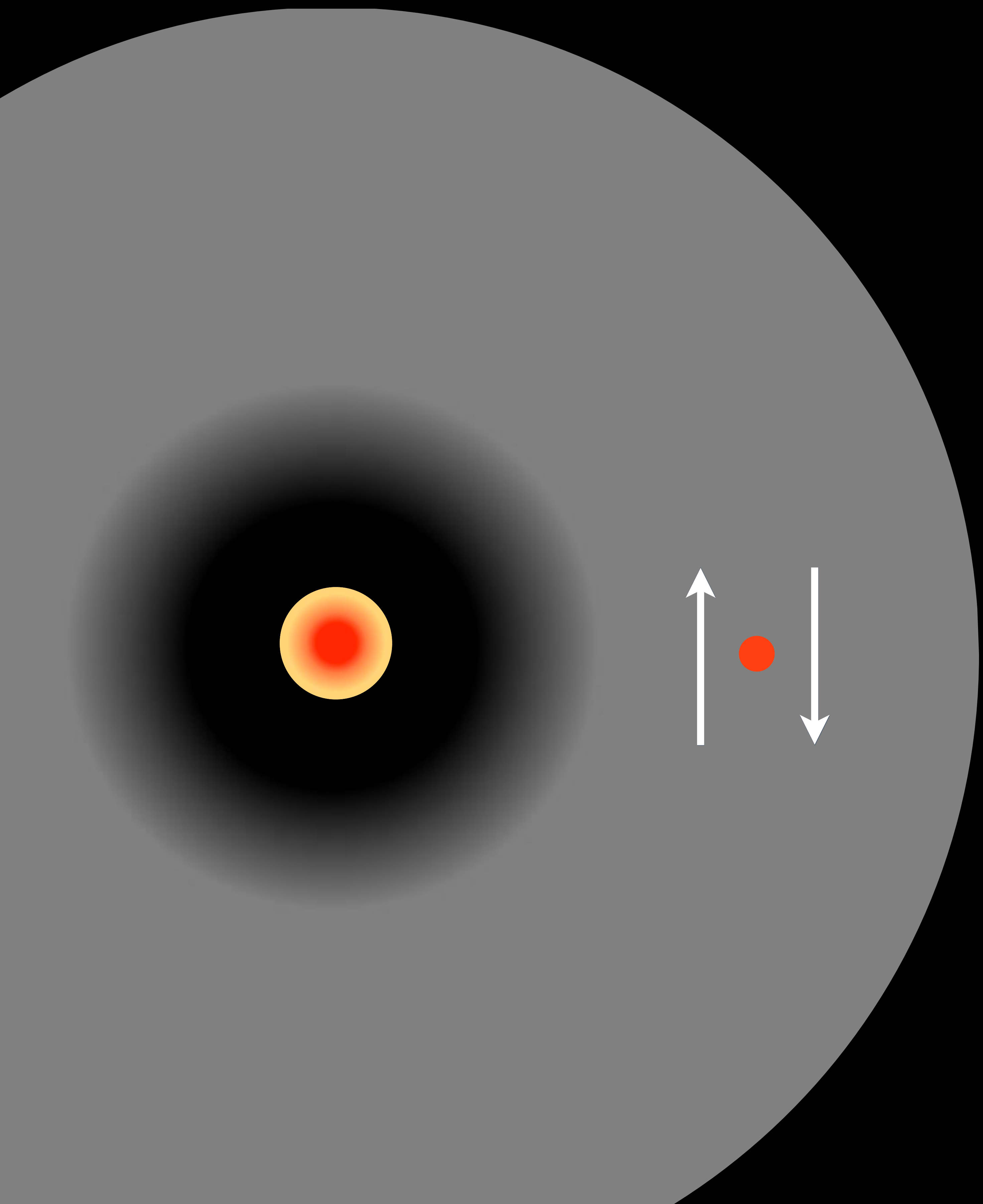


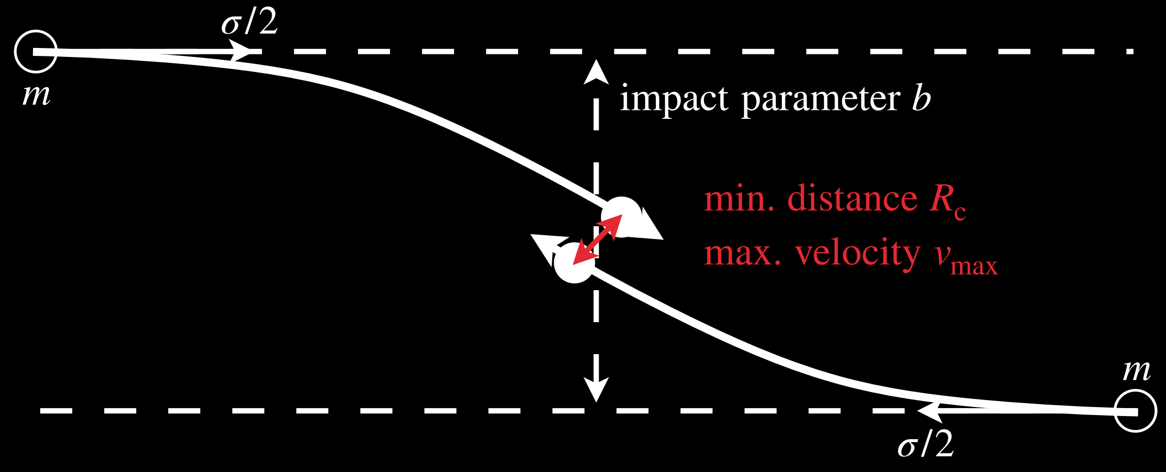
$\sigma/2$
m
impact parameter b
min. distance R_c
max. velocity v_{max}
m
$\sigma/2$



$2\Delta a$
ϕ
r
$\Delta a \approx 2.3 r_H$
$\frac{\Delta a}{2}$
Δa
$\frac{\Delta a}{2}$
too far away
just right
horseshoe
just right
too far away

REVIEW





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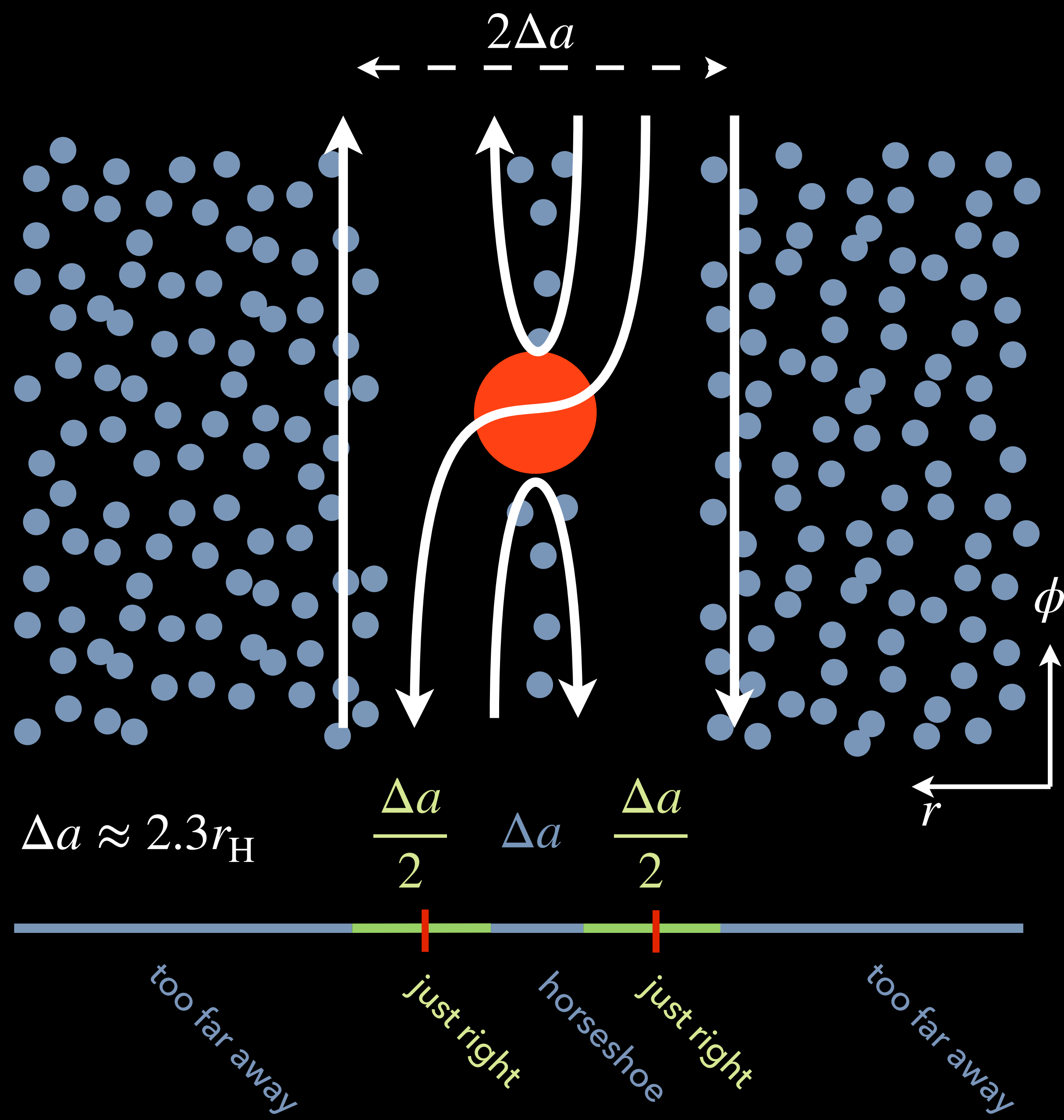

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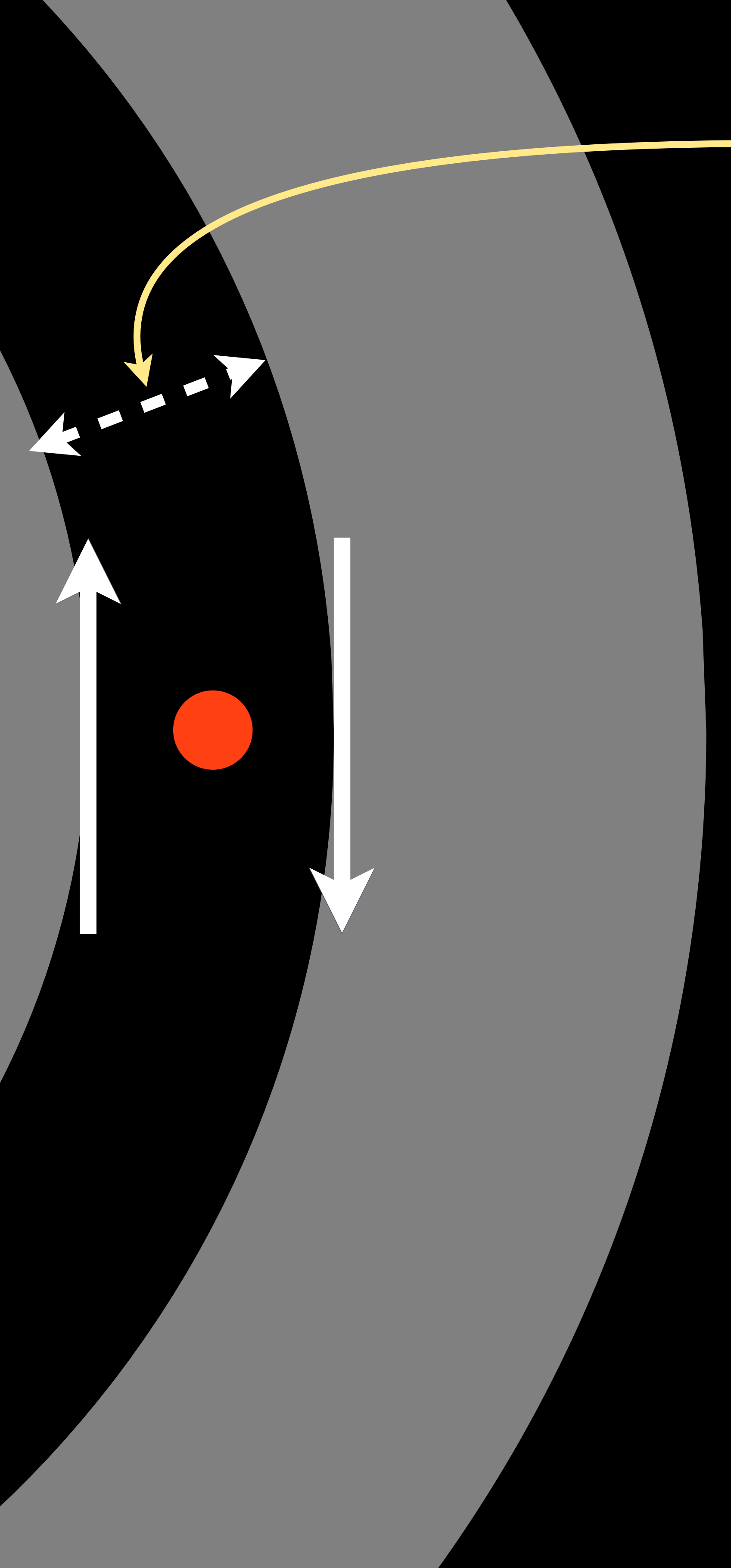
REVIEW



width scales with hill radius: $r_{\mathrm{H}} = a_{\mathrm{p}} \sqrt[3]{\frac{GM_{\mathrm{p}}}{3M_*}}$

$$\Delta a_{\max} \approx C r_{\mathrm{H}}$$

Mass in the feeding zone grows with planet mass:

$$M_{\mathrm{fz}} \approx 2\pi a_{\mathrm{p}} \cdot 2\Delta a_{\max} \cdot \Sigma_{\mathrm{p}} \propto M^{1/3}$$

Isolation mass grows with cylindrical radius:

$$M_{\mathrm{iso}} = \frac{8}{\sqrt{3}} \pi^{3/2} C^{3/2} M_*^{-1/2} \Sigma^{3/2} a_{\mathrm{p}}^3$$

$M_{\mathrm{iso}} \approx 0.07\ M_\oplus$ in the terrestrial region

$M_{\mathrm{iso}} \approx 9\ M_\oplus$ in the giant planet region

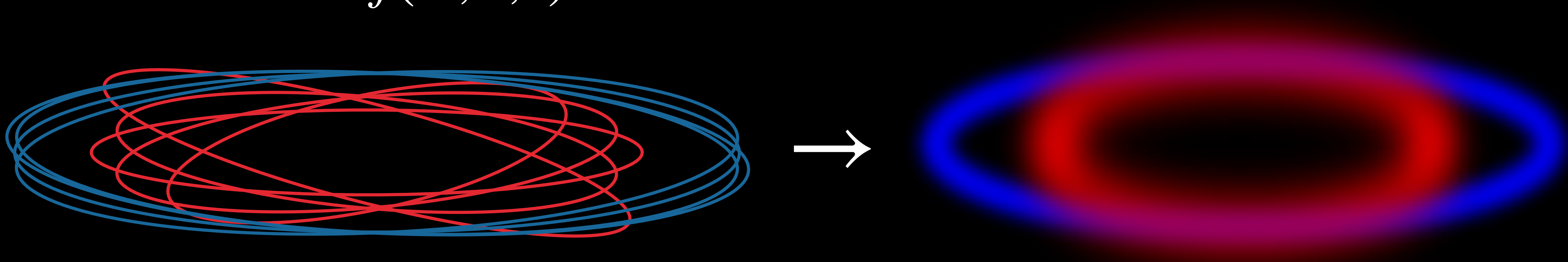
FROM UNIVERSE

TO PLANETS

LECTURE 4.1: PLANETESIMAL INTERACTIONS

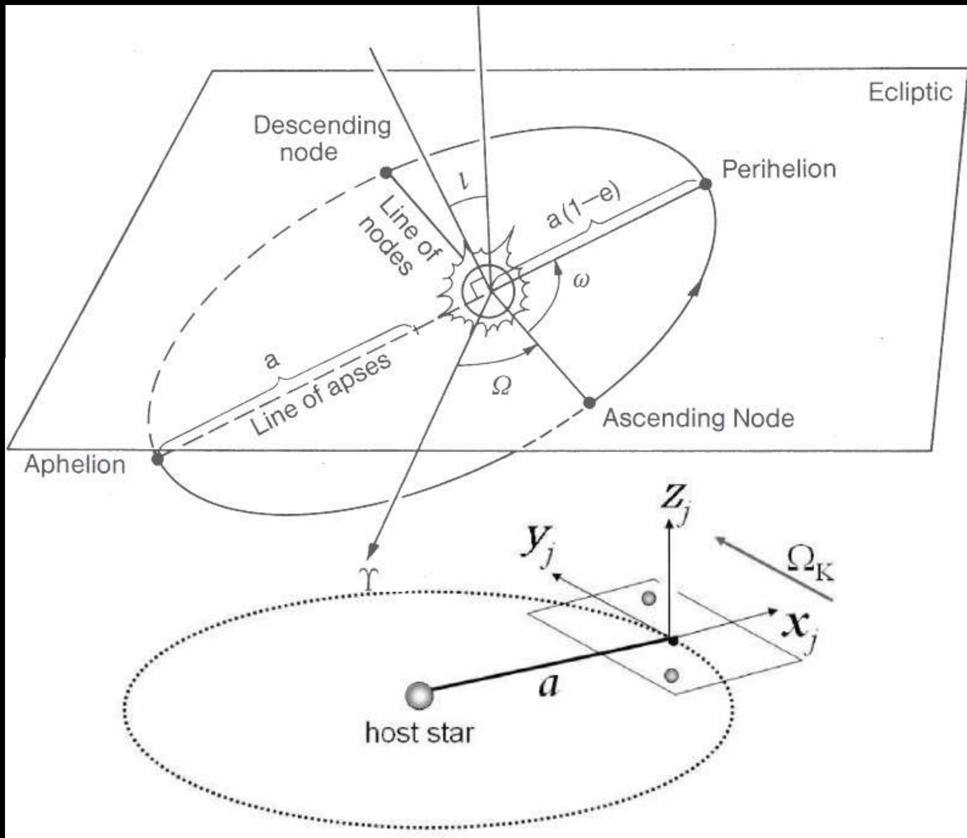
RANDOM VELOCITIES

- Large number of planetesimals: statistical treatment
- Similar to gas molecules and kinetic gas theory
- "Hot" (many collisions, high velocity) vs. "cold" distributions
  - "Heating": mutual gravitational scattering
  - "Cooling": collisions, gas drag, ejection
- Treat eccentricity e, inclination i, and mass m using a distribution $f(m, e, i)$



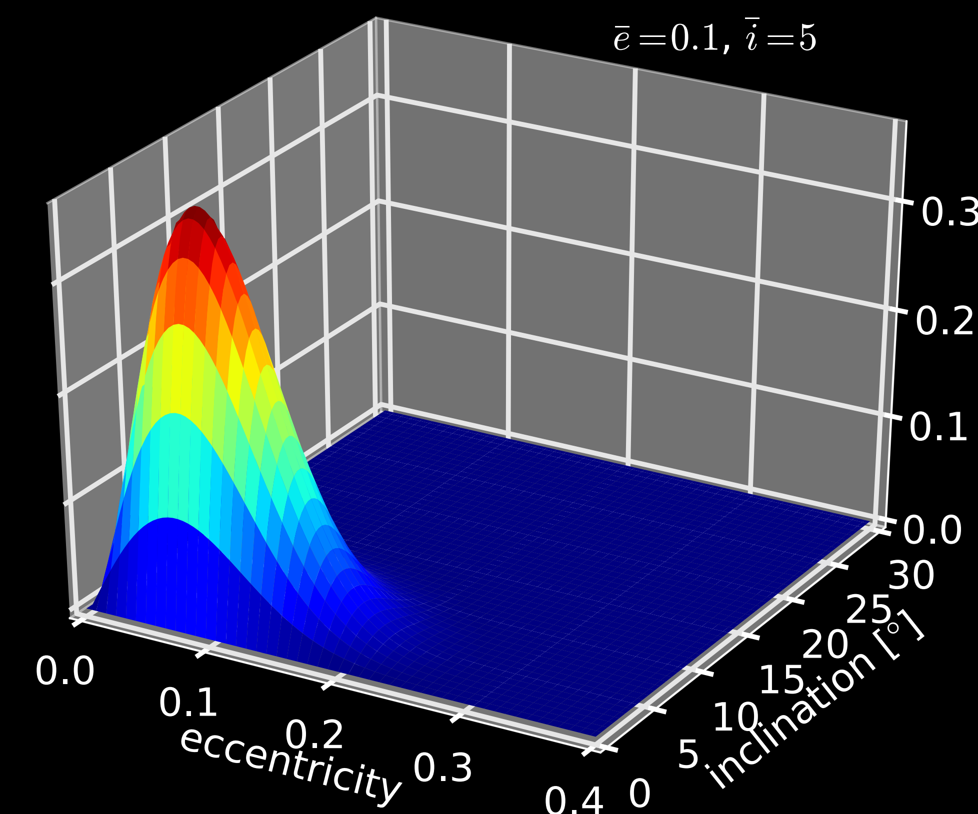
RANDOM VELOCITIES

- Kepler motion is characterised by 6 orbital elements:
  - Semi-major axis: $a_{\rm p}$
  - Eccentricity: e
  - Inclination: i
  - Longitude of ascending node: Ω
  - Longitude of perihelion: ω
  - Time of passage at perihelion: T



- Hill coordinates (x, y, z) are rotating coordinates with the Keplerian angular velocity. In this frame, Keplerian motion is a superposition of shear, epicyclic, and vertical oscillations.
- For random Ω, ω, T the oscillations are randomly distributed about the systematic shear flow: $v \approx \sqrt{e^2 + i^2} v_{\rm K}$

RANDOM VELOCITIES



- Numerical simulations give Rayleigh distribution for e and i:

$$f(e,i) = \frac{4e\sin i}{\langle e^2\rangle\langle \sin^2 i\rangle} \exp\left[-\frac{e^2}{\langle e^2\rangle} - \frac{\sin^2 i}{\langle \sin^2 i\rangle}\right]$$

for small angles: $\sin i \sim i$

- Gaussian distribution of the random components v_R, v_z, and $\delta v_\phi = v_\phi - v_\mathrm{K}$ with velocity dispersions:

$$\sigma_R^2 = \frac{1}{2}\langle e^2\rangle v_\mathrm{K}^2 \qquad \sigma_z^2 = \frac{1}{2}\langle i^2\rangle v_\mathrm{K}^2$$

RMS values: $\langle \cdot \rangle$

- Equipartition of energy between eccentric and inclined motions (isotropic distribution with an equilibrium between "heating" and "cooling"): $\sqrt{\langle e^2\rangle} = 2\sqrt{\langle i^2\rangle}$

RANDOM VELOCITIES

- The Jacobi integral E_J is the only conserved quantity for the circular restricted three-body problem (the energy and momentum of the system are not separately conserved). For $m \ll M_p \ll M_*$ that are not too close:

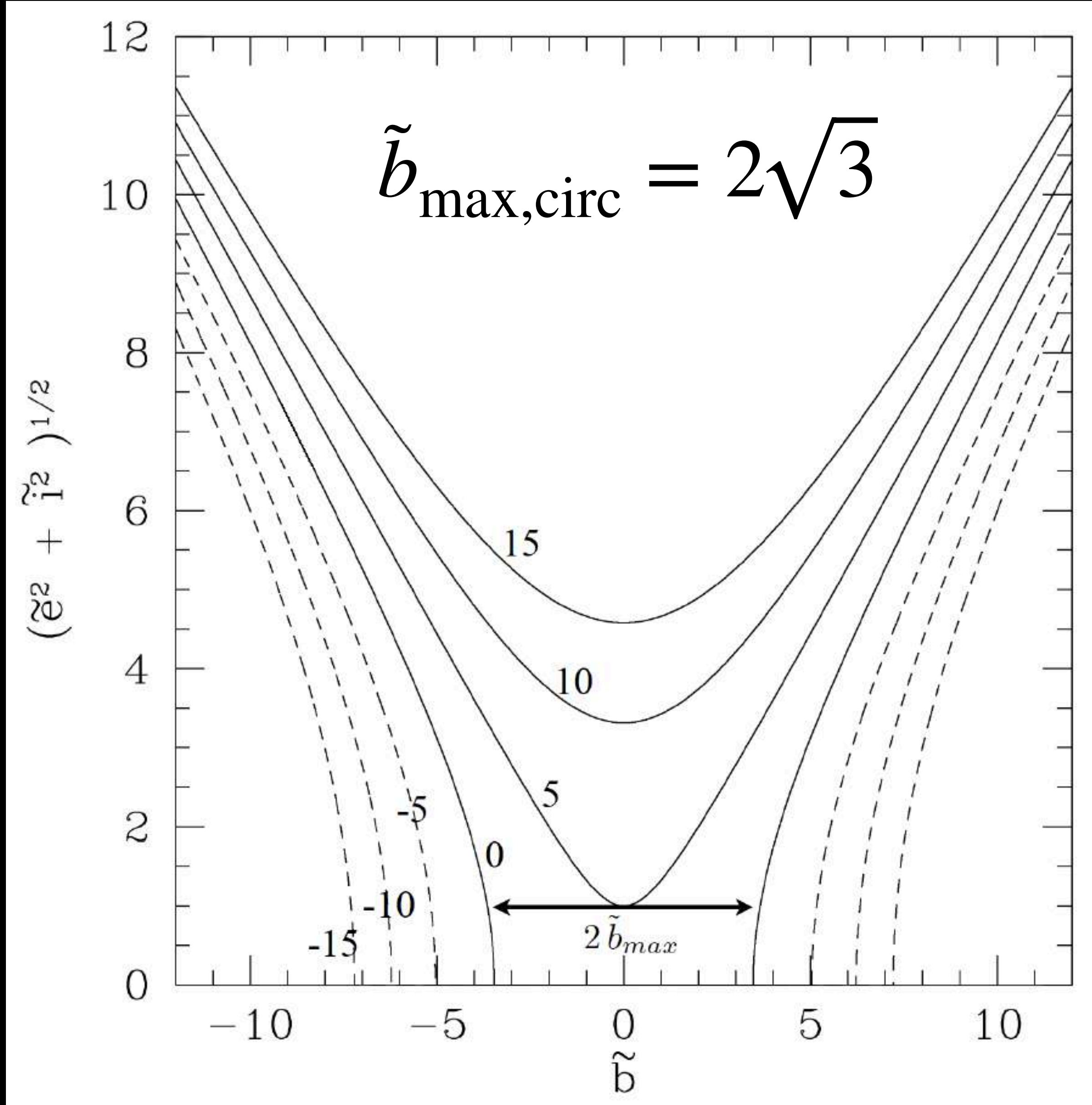
$$\tilde{E}_\mathrm{J} = \frac{1}{2}\left(\tilde{e}^2 + \tilde{i}^2\right) - \frac{3}{8}\tilde{b}^2 + \frac{9}{2}$$

$$\tilde{e} = \frac{e}{h} \qquad \tilde{i} = \frac{i}{h} \qquad \tilde{b} = \frac{b}{a_\mathrm{p} h}$$

$$h = \left(\frac{m}{3M_*}\right)^{1/3} \qquad \tilde{E}_\mathrm{J} = \frac{E_\mathrm{J}}{a_\mathrm{p}^2 h^2 \Omega_\mathrm{K}^2}$$

- The region $\tilde{E}_\mathrm{J} > 0$ defines the feeding zone of the larger mass:

$$2\tilde{b}_\mathrm{max} = 2\left[\frac{4}{3}\left(\tilde{e}^2 + \tilde{i}^2\right) + 12\right]^{1/2}$$

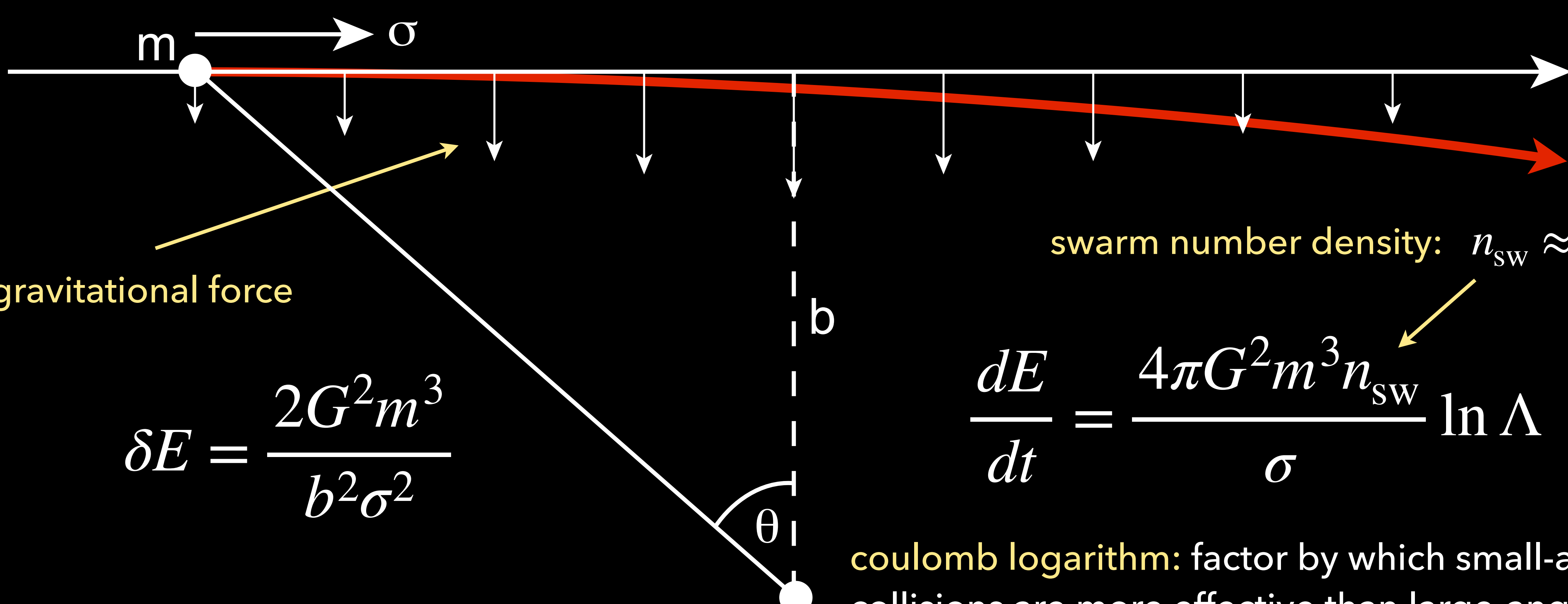


RANDOM VELOCITIES

- The random velocities set the growth regime: orderly, runaway, or oligarchic. The key ingredients are:
    - Viscous stirring (increase of random velocities) through collisions or gravitational scattering between planetesimals and protoplanets.
    - Dynamical friction: energy transfer from large to small bodies
        - Establishes energy equipartition between bodies of different sizes.
    - Damping due to dissipation in inelastic collisions and through gas drag.

VISCOUS STIRRING

- Distant fly-bys produce small angle deflections.
- Transforms some of the forward velocity (kinetic energy) to a random perpendicular velocity/energy.
- Integration over all encounters gives the increase of the random kinetic energy.



$$\delta E = \frac{2G^2m^3}{b^2\sigma^2}$$

swarm number density: $n_{sw} \approx \frac{\Sigma_{sw}\Omega_K}{2m\sigma}$

$$\frac{dE}{dt} = \frac{4\pi G^2 m^3 n_{sw}}{\sigma} \ln \Lambda$$

coulomb logarithm: factor by which small-angle collisions are more effective than large-angle collisions

VISCOUS STIRRING

- The rate of increase of the random velocity

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{1}{2}m\sigma^2\right) \quad \longrightarrow \quad \frac{4\pi G^2 m^3 \left(\frac{\Sigma_{\rm sw}\Omega_{\rm K}}{2m\sigma}\right)}{\sigma}\ln\Lambda = m\sigma\frac{d\sigma}{dt}$$

$$\frac{d\sigma}{dt} = \frac{2\pi G^2 m}{\sigma^3}\Sigma_{\rm sw}\Omega_{\rm K}\ln\Lambda \propto \frac{1}{\sigma^3} \quad \longrightarrow \quad \sigma \propto t^{1/4}$$

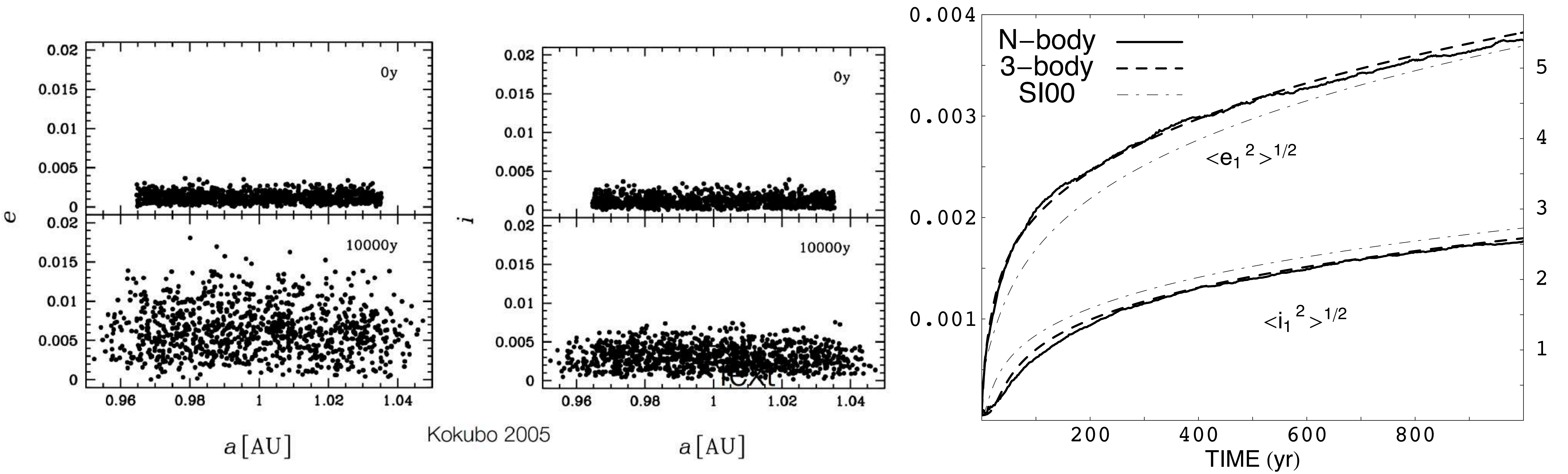
- Relaxation timescale: the time it takes for a particle to forget its initial velocity (or orbit).

$$\tau_{\rm relax} = \frac{\sigma^2}{d\sigma^2/dt} = \frac{2\sigma^4}{\Sigma_{\rm sw}\Omega_{\rm K}\pi G^2 m\ln\Lambda}$$

- The viscous stirring rate is related to $\tau_{\rm relax}$ by: $\tau_{\rm vs} = \frac{d\langle e^2\rangle}{dt} \sim \frac{\langle e^2\rangle}{\tau_{\rm relax}}$
- $\tau_{\rm vs} \sim 6000$ yrs for typical values at 1 au and $m = 10^{19}$ g.

VISCOUS STIRRING

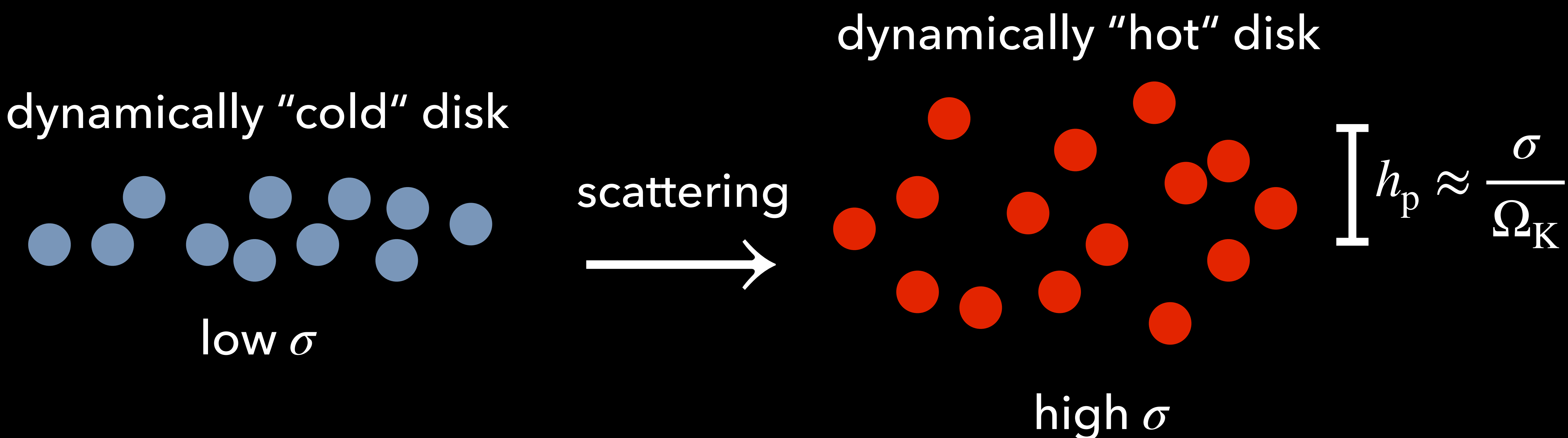
- On average, the increase of e is larger than that of i, but both relax into a Rayleigh distribution.
- The diffusion of planetesimals in semi-major axis is a result of a random walk due to two-body scattering.
- The growth rate of a protoplanet tends to decrease as it "heats" the neighbouring planetesimals.



Kokubo 2005

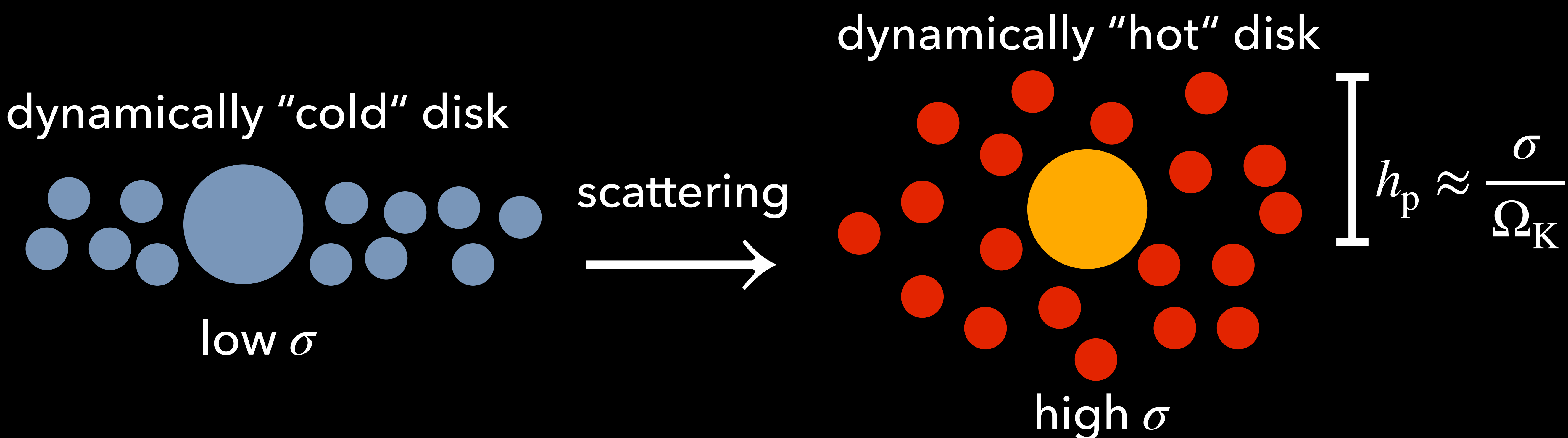
DYNAMICAL FRICTION

- Same principle as before, but now with different masses.



DYNAMICAL FRICTION

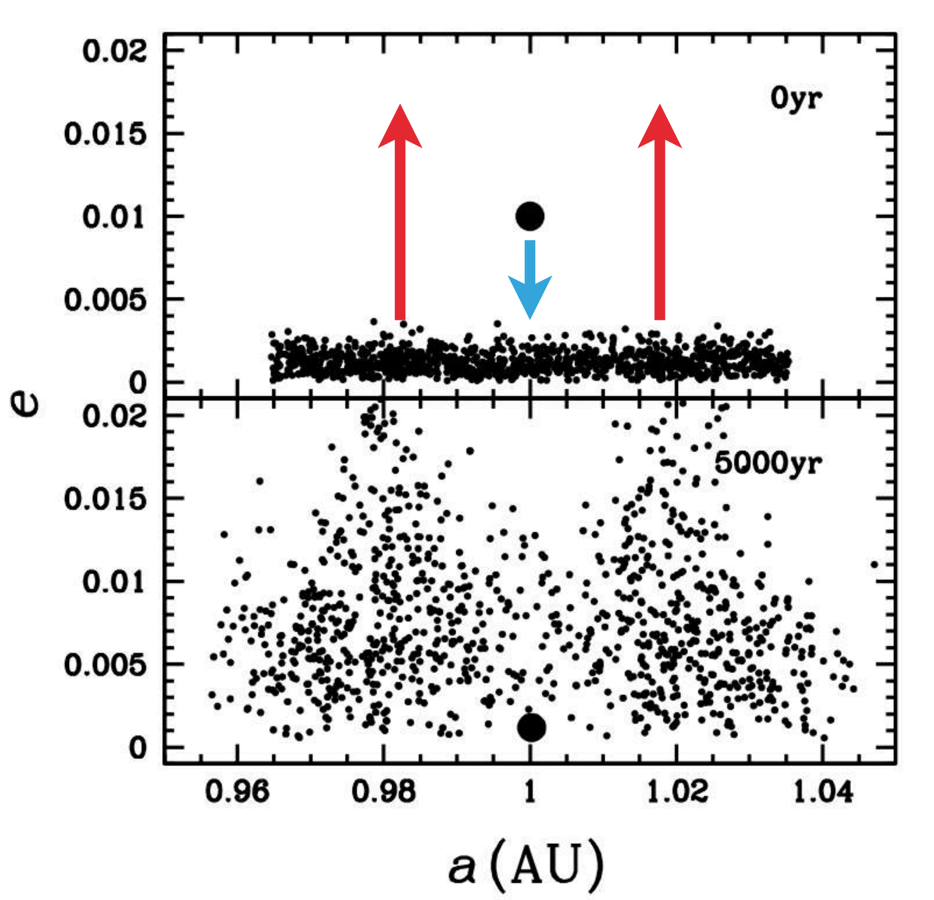
- Same principle as before, but now with different masses.
- Massive body heats up its environment

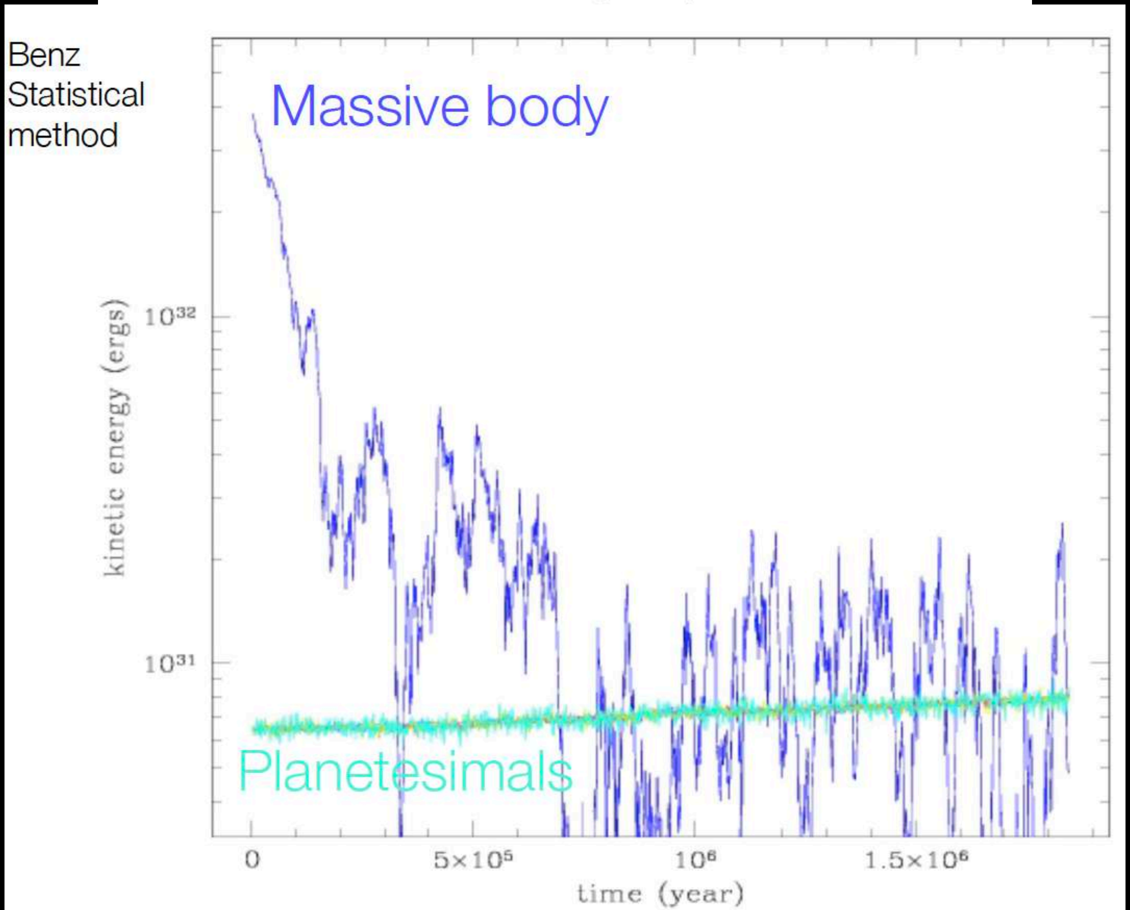


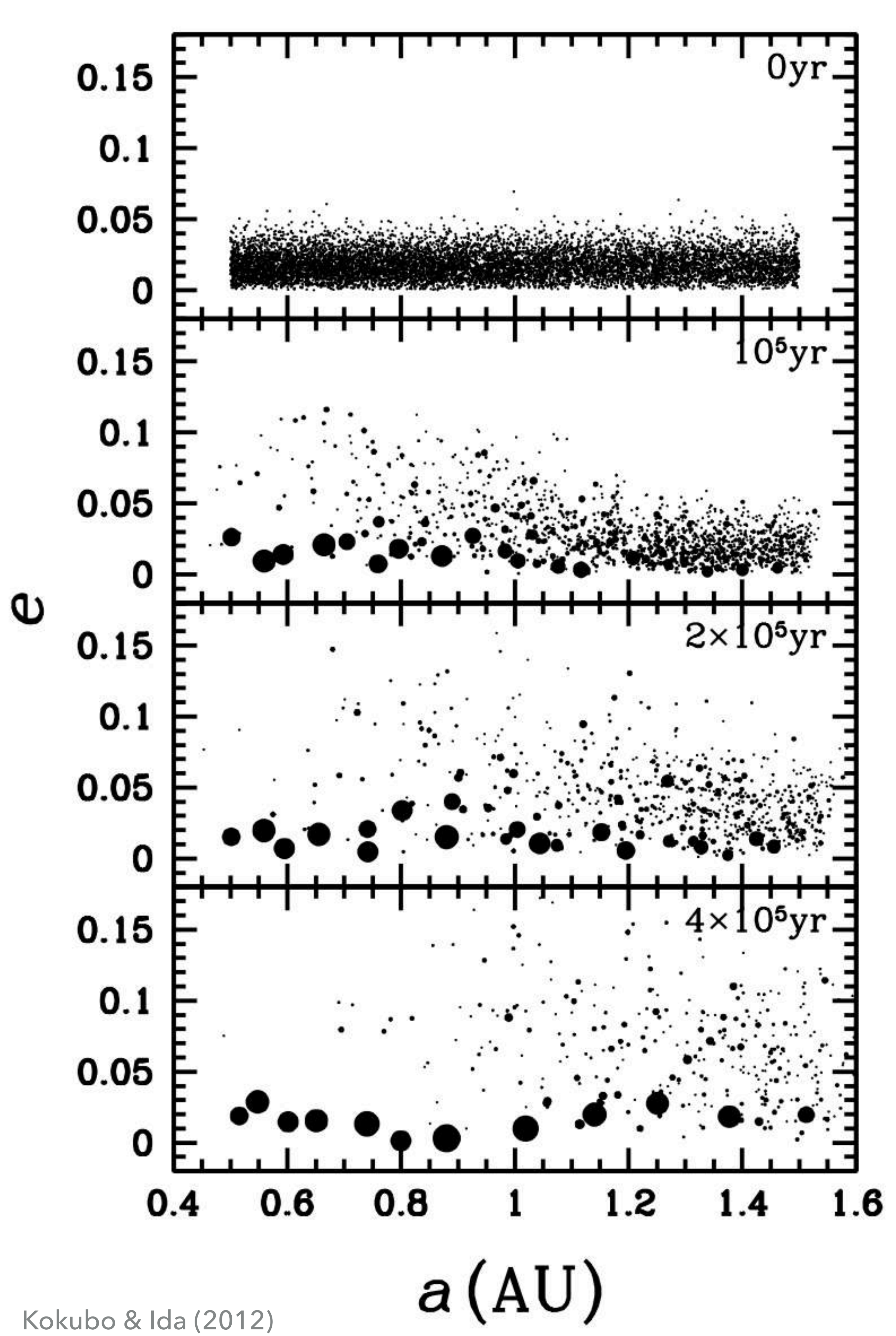
Massive body is scattered less, smaller masses are scattered more. Many encounters lead to energy equipartition:

$$\frac{1}{2} m \sigma_m^2 = \frac{1}{2} M \sigma_m^2$$

DYNAMICAL FRICTION



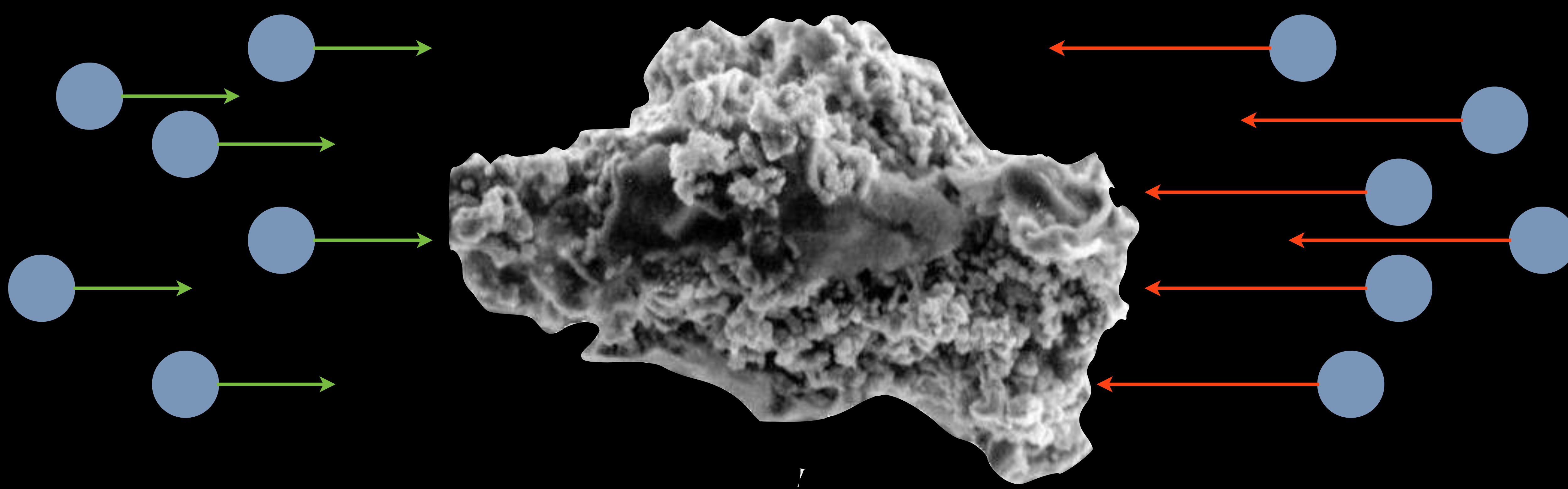




Kokubo & Ida (2012)

GAS DAMPING

- Gas drag is similar to that of smaller particles, but sizes are large enough to put them in the hydrodynamic (Stokes) regime.



$$F_{\text{Epstein}} = -\frac{4\pi}{3}\rho_{\text{gas}} a^2 v_{\text{th}} \mathbf{v}$$

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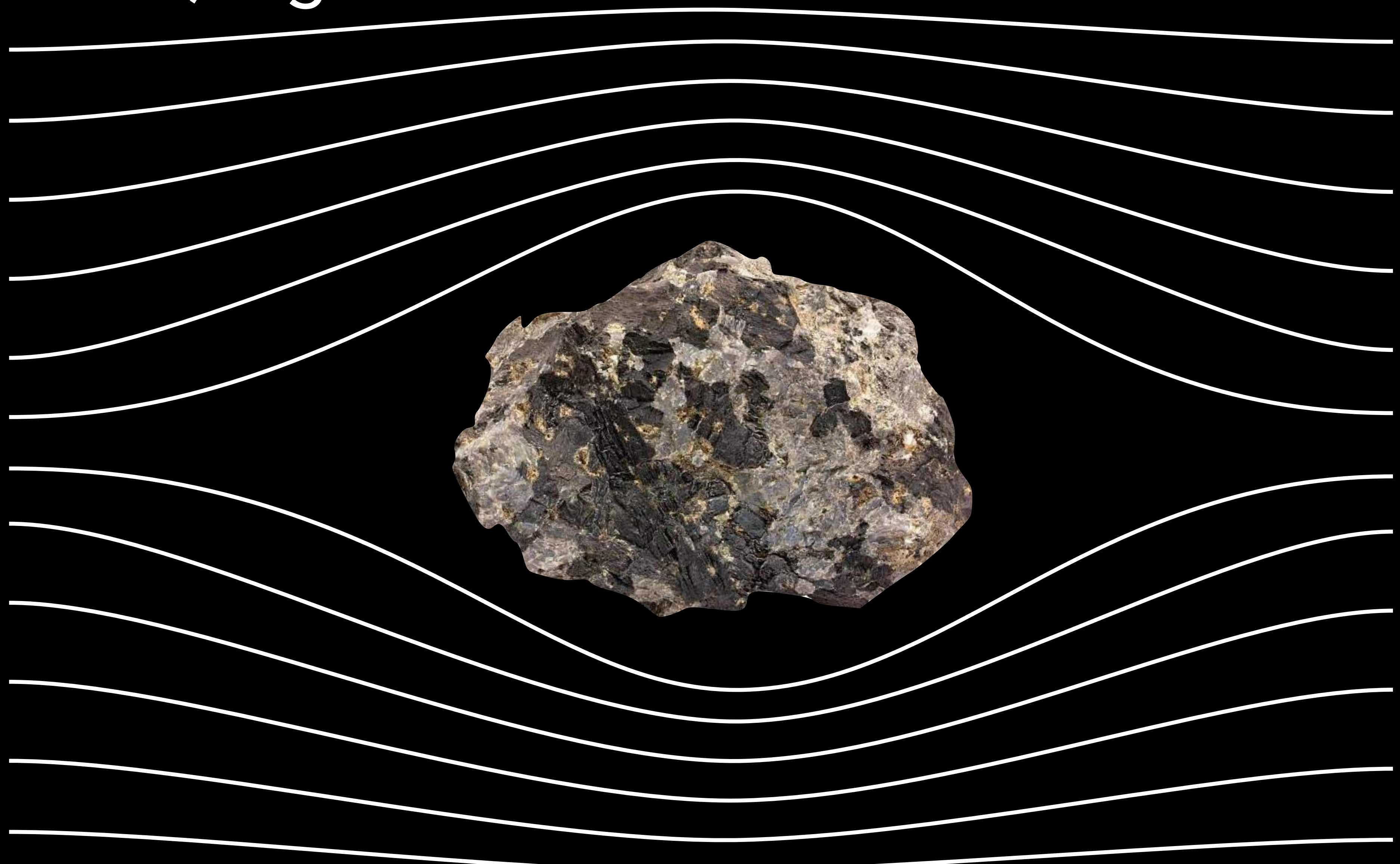
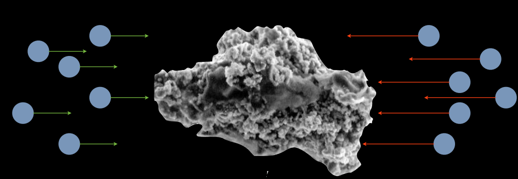


$$F_{\text{Stokes}} = -\frac{C_{\text{D}}}{2}\pi a^2 \rho_{\text{gas}} v \mathbf{v}$$



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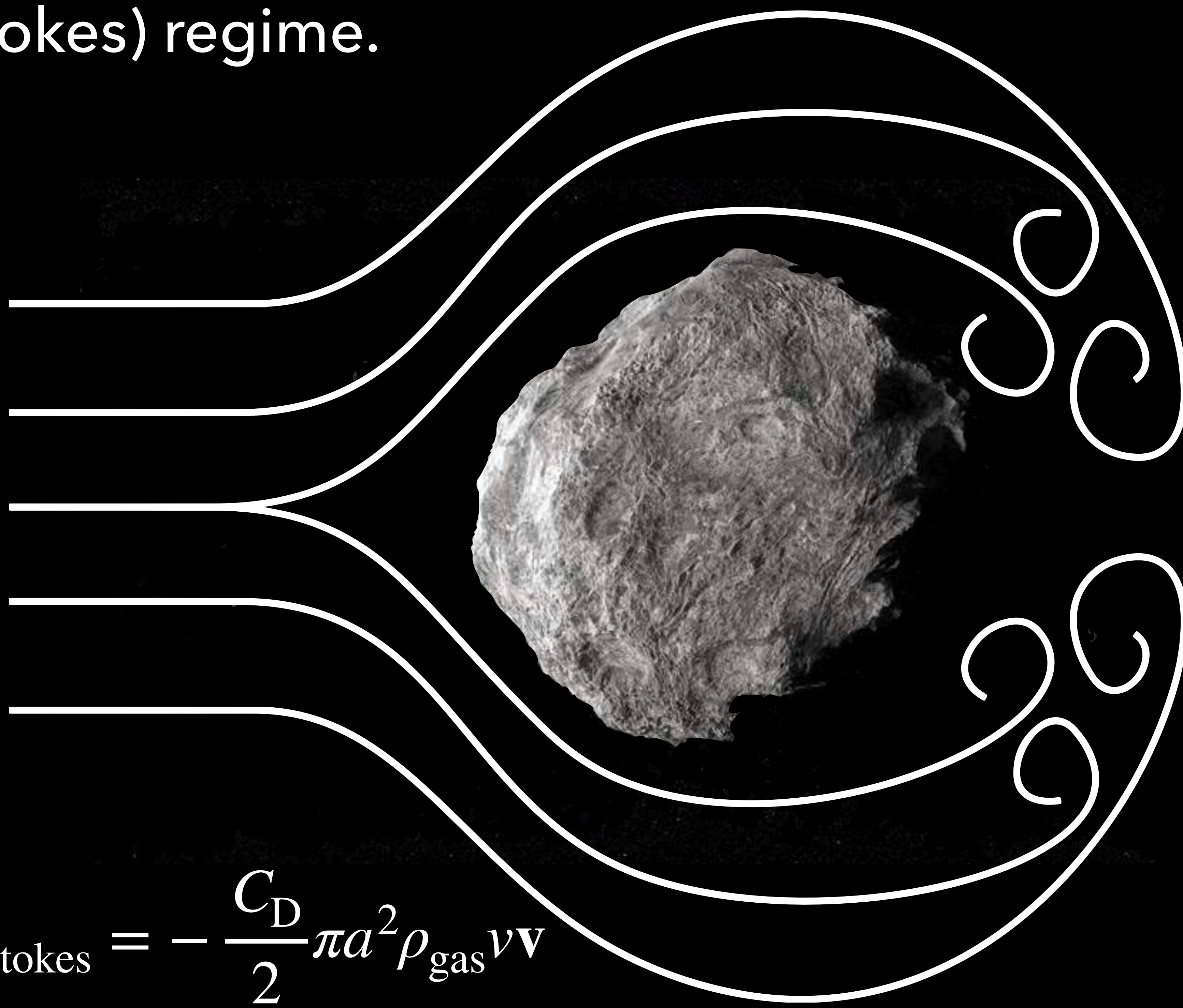

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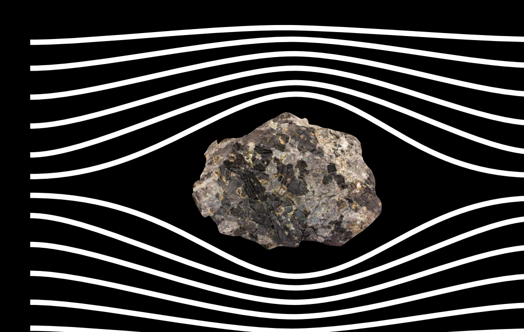


GAS DAMPING

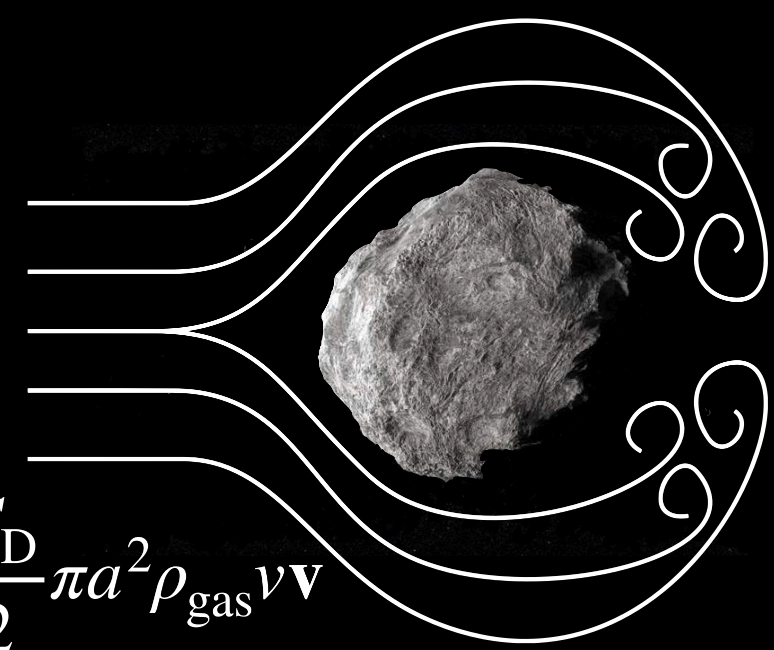
- Gas drag is similar to that of smaller particles, but sizes are large enough to put them in the hydrodynamic (Stokes) regime.

- Damping acts against viscous stirring, thereby facilitating growth. For strong damping, the system evolves into a shear dominated regime (thin planetesimal disc). The 2D dynamics lead to high collisional probabilities and large focussing factors $\longrightarrow$ large growth rates.

- Assuming the random velocity is $v \sim e v_\mathrm{K}$, the damping timescales is approximately:

$$\tau_\mathrm{damp,gas} \approx \frac{mv}{F_\mathrm{drag}} = \frac{2m}{C_\mathrm{D} \pi a^2 \rho_\mathrm{gas} e v_\mathrm{K}} \propto a$$

$$F_\mathrm{Stokes} = -\frac{C_\mathrm{D}}{2} \pi a^2 \rho_\mathrm{gas} v \mathbf{v}$$



ORDERLY AND RUNAWAY GROWTH

- Recall our mass accretion rate including the focusing factor:

$$\frac{dM_{\rm p}}{dt} = \frac{\sqrt{3}}{2}\Sigma_{\rm p}\Omega_{\rm K}\,\pi R_{\rm s}\overbrace{\left(1+\frac{v_{\rm esc}^2}{\sigma^2}\right)}^{\Gamma=\text{focusing factor}}$$

- Depending on the velocity dispersion, we get two different growth regimes:

$$\frac{1}{M_{\rm p}}\frac{dM_{\rm p}}{dt} \propto \begin{cases} M_{\rm p}^{-1/3}, & \sigma \gg v_{\rm esc}\ \text{(orderly)} \\ M_{\rm p}^{1/3}, & \sigma \ll v_{\rm esc}\ \text{(runaway)} \end{cases}$$

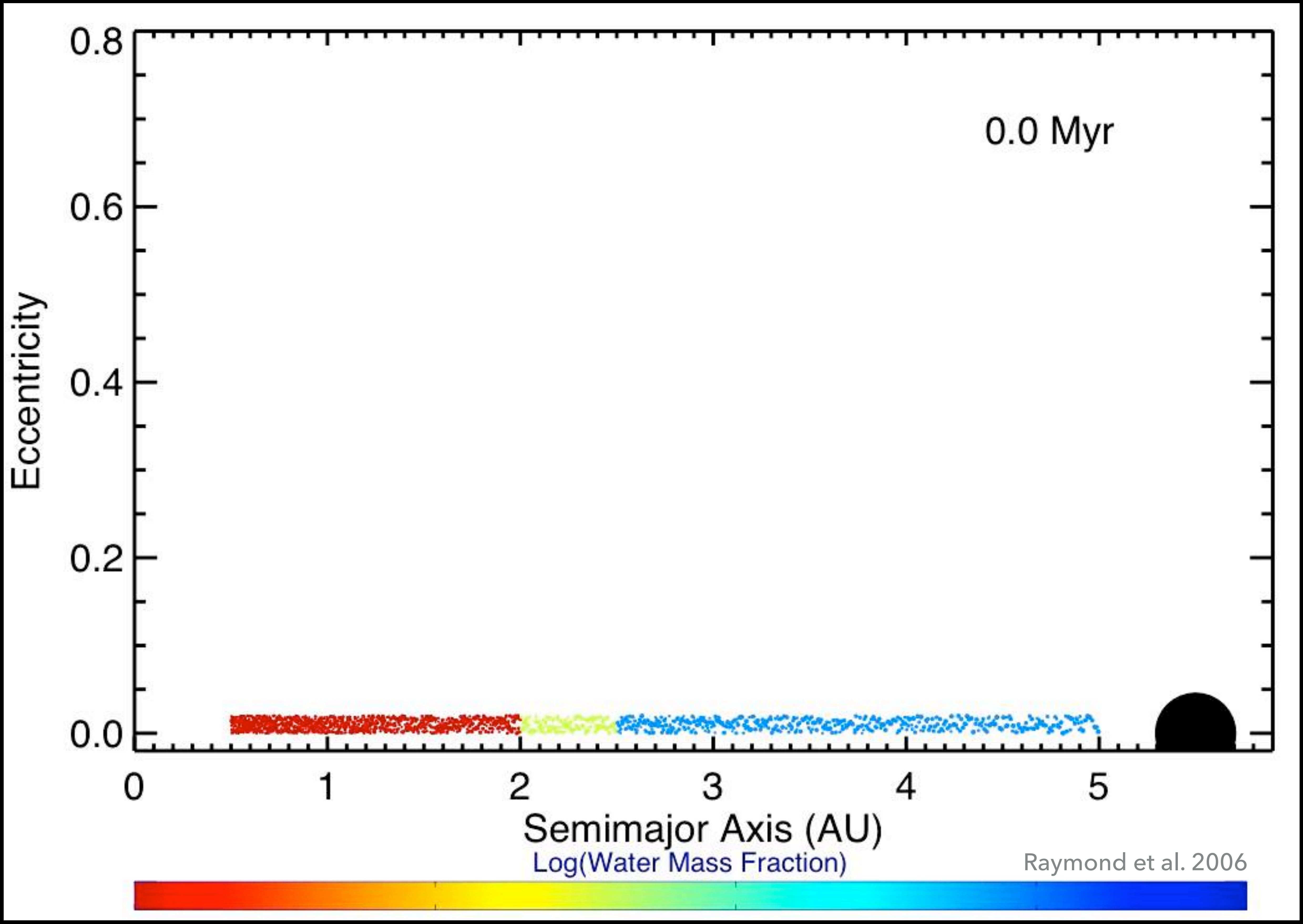
- Viscous stirring increases σ, leading to orderly growth, with a power-law size distribution having most of the mass in the largest bodies.

- Addition of Dynamical friction and gas drag tends to equalise kinetic energies and damp σ of the more massive bodies, leading to runaway growth of a small number of embryos.

OLIGARCHIC GROWTH

- Runaway growth continues until the feedback from the big bodies stirs up the neighbouring planetesimals again.
- Growth rates slows (similar to orderly growth), but are still faster than planetesimals in their surroundings (similar to runaway growth).
- Transition from runaway to oligarchic growth occurs between 10^{-3}–$10^{-2}\ M_{\oplus}$

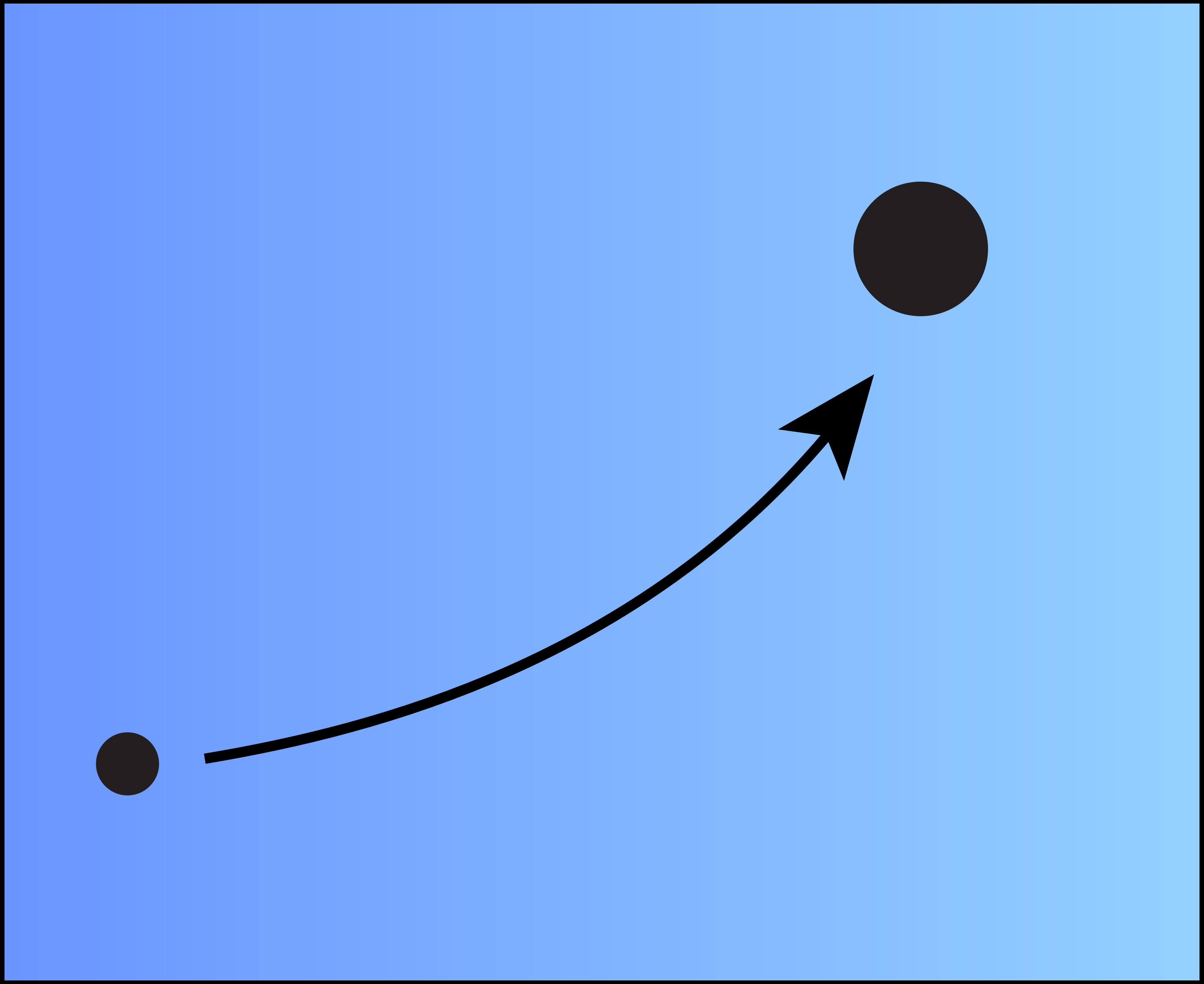
OLIGARCHIC GROWTH



Raymond et al. 2006

GAS ACCRETION

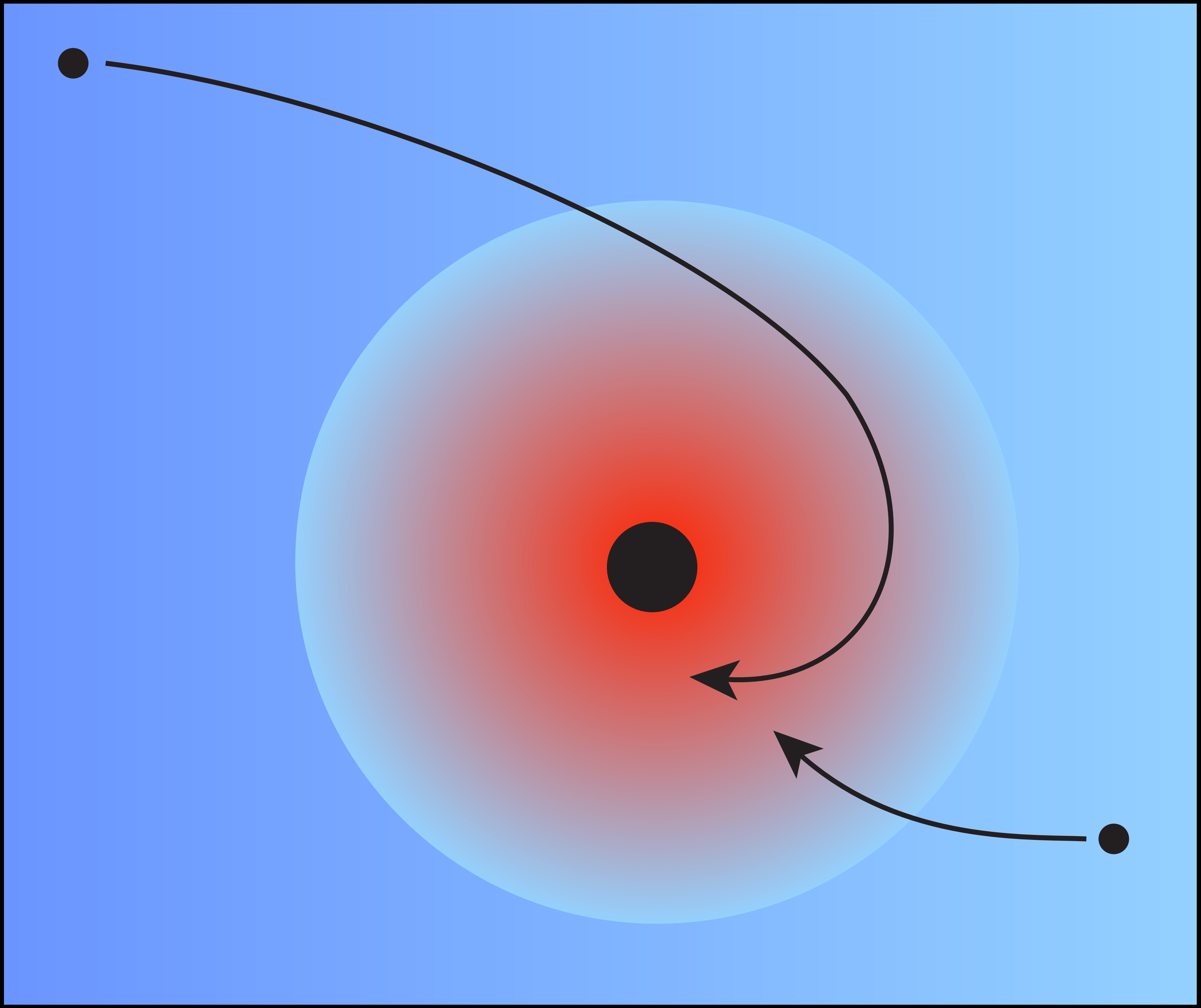
1. Core formation



2. Atmo

GAS ACCRETION

3. Hydrostatic growth

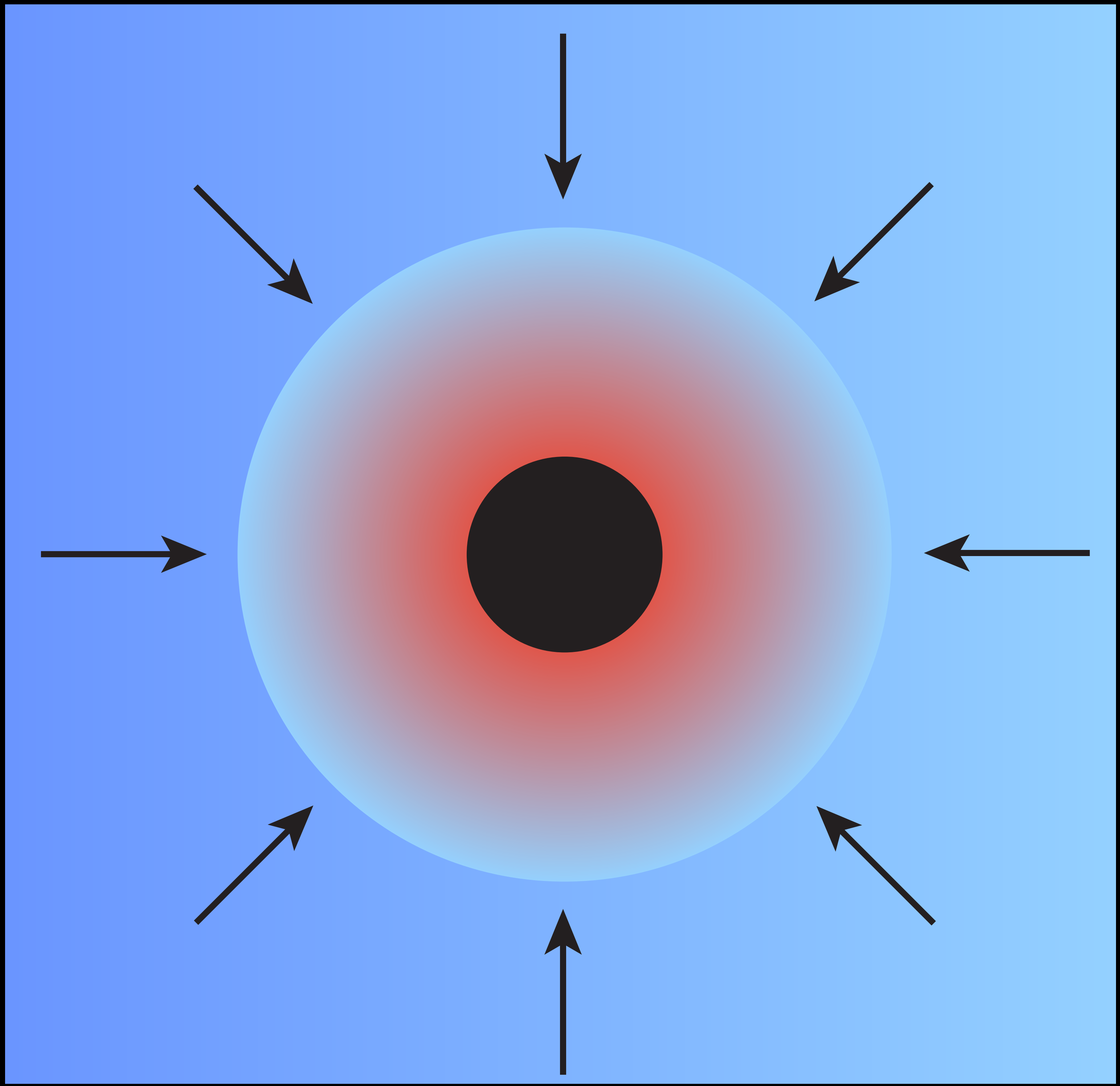


rmation

4. Exceed

GAS ACCRETION

5. Runaway accretion

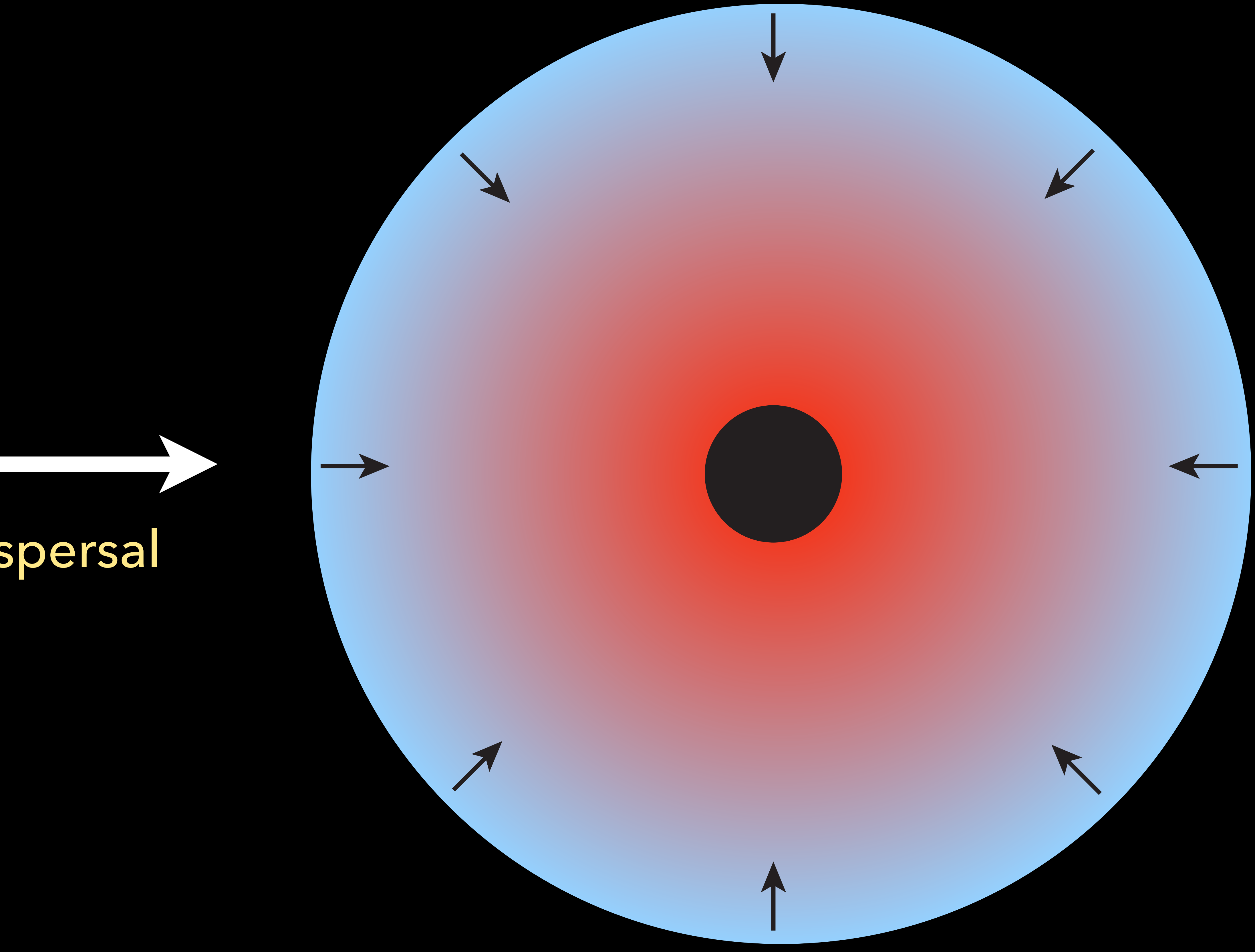


l core mass

6. G

GAS ACCRETION

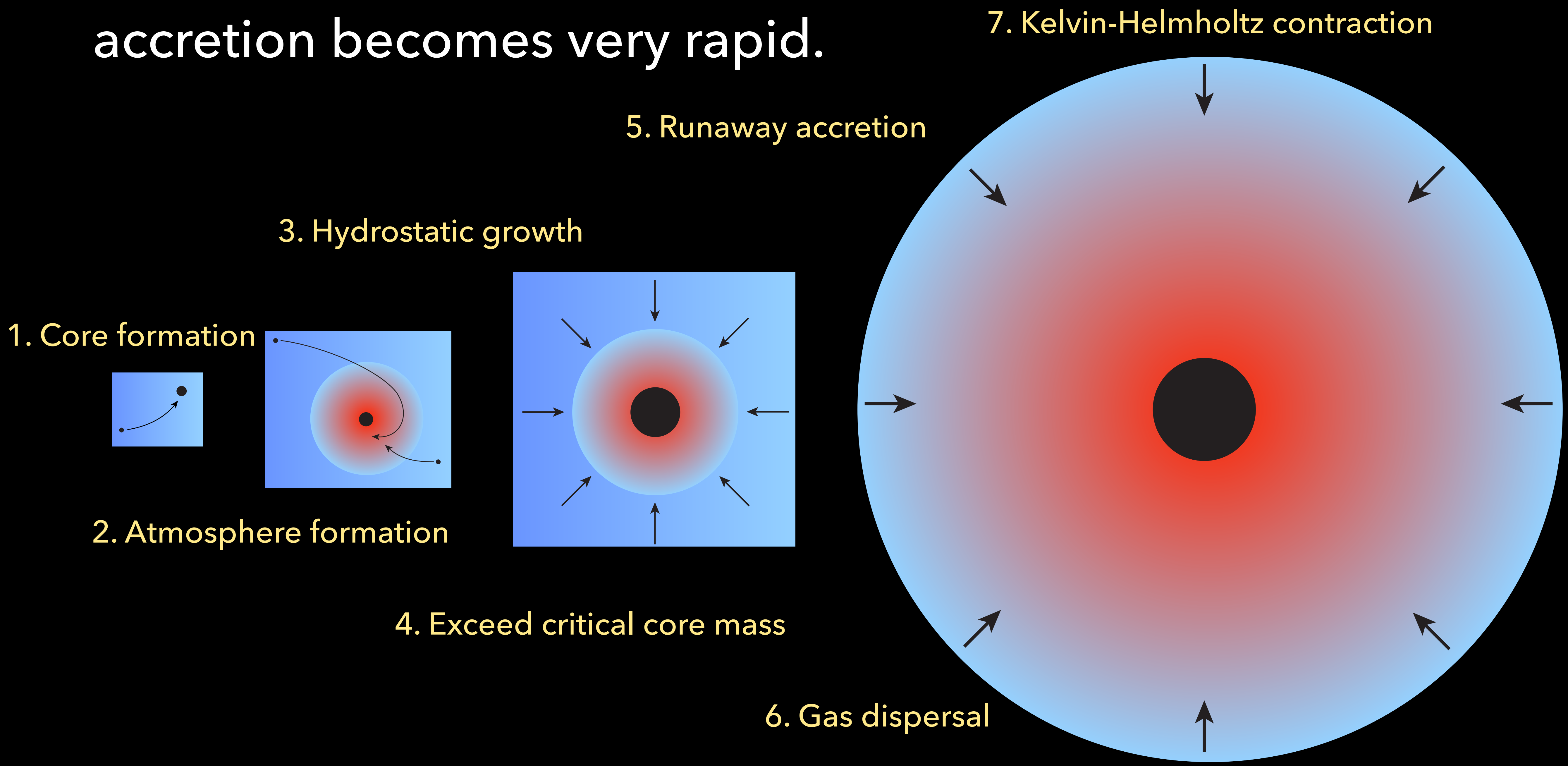
7. Kelvin-Helmholtz contraction



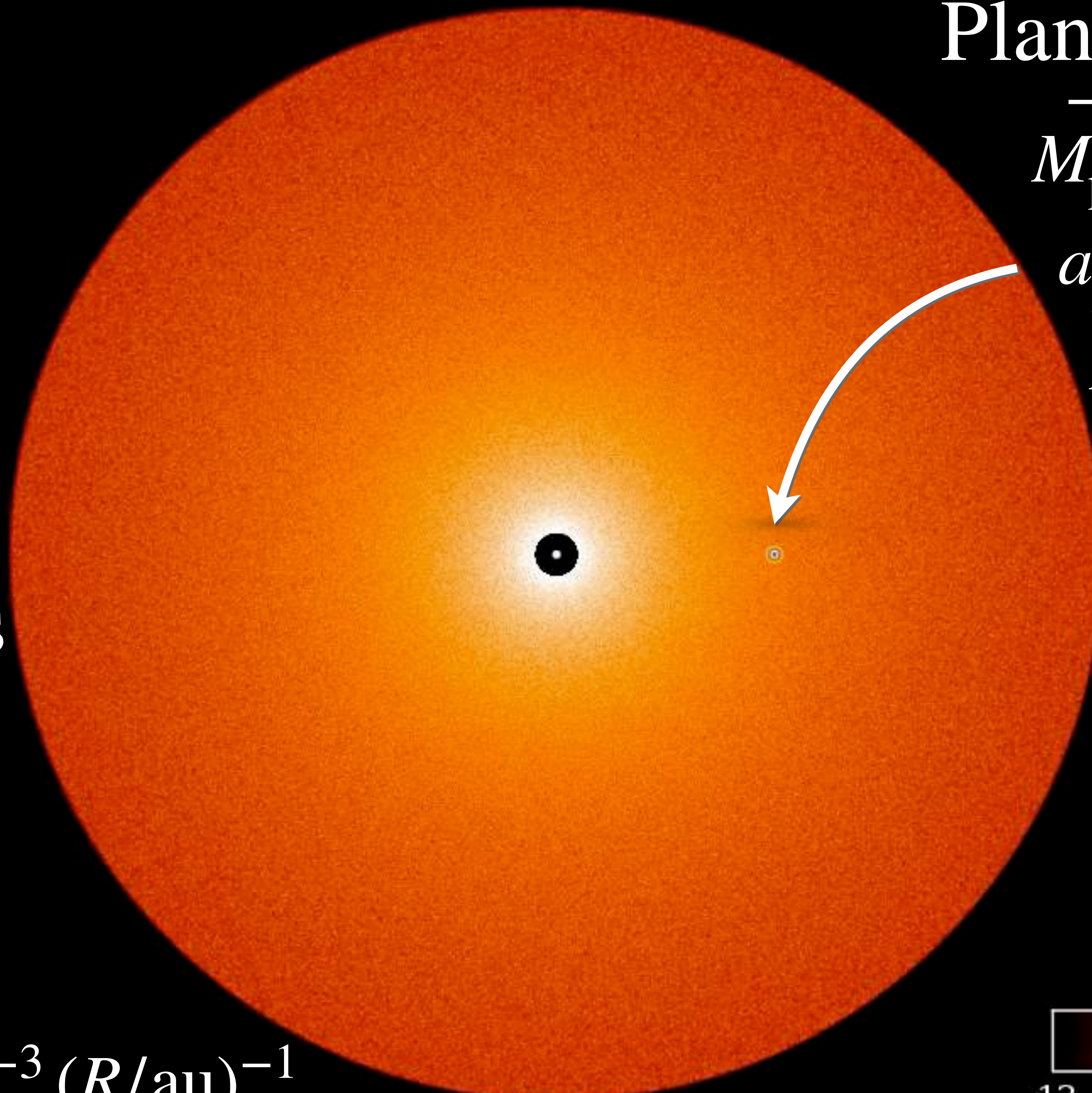
spersal

GAS ACCRETION

- Gas accretion begins very slowly because it is pressure supported. Prevents new gas from being accreted.
- Once a critical core mass is achieved, runaway gas accretion becomes very rapid.



GAS ACCRETION: EMBEDDED PHASE



Planet properties

$M_{\rm planet} = 15\, M_\oplus$

$a_{\rm planet} = 0.25$ au

$R_{\rm core} \approx 0.00011$ au

Disc properties

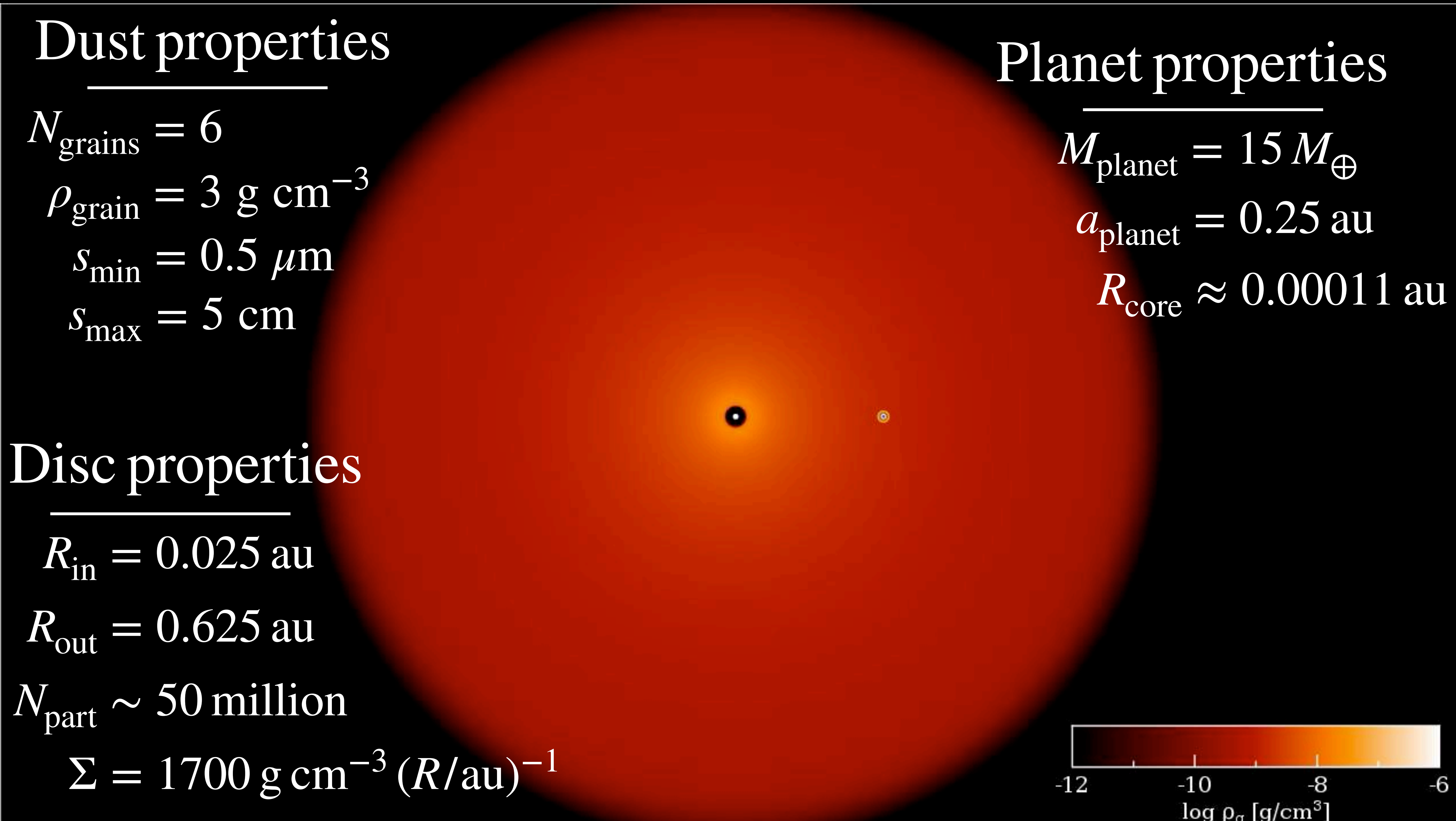
$R_{\rm in} = 0.025$ au

$R_{\rm out} = 0.625$ au

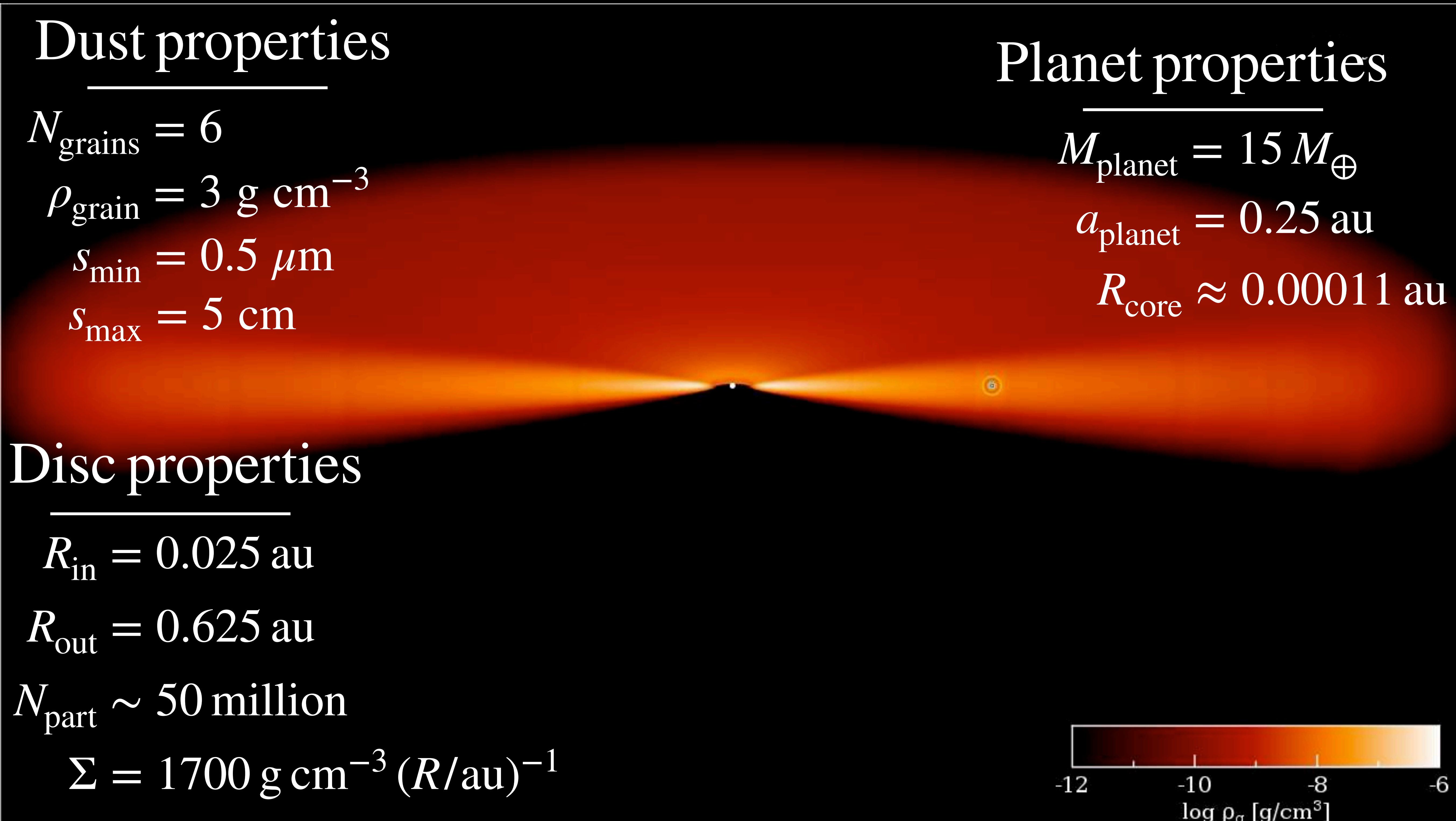
$N_{\rm part} \sim 50$ million

$\Sigma = 1700\ {\rm g\,cm^{-3}}\,(R/{\rm au})^{-1}$

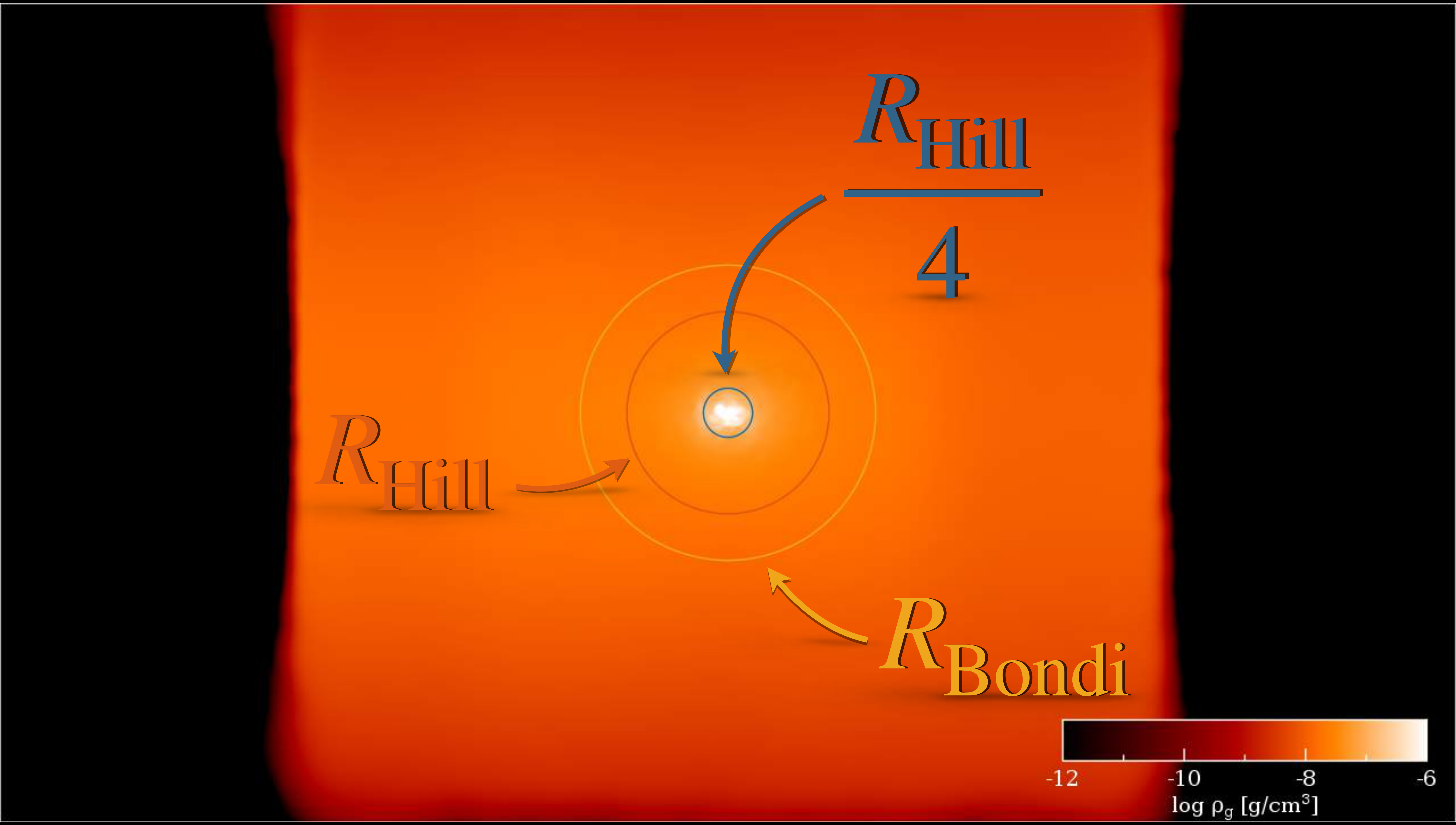
Mid-plane cross section


Dust properties
$N_{\rm grains} = 6$
$\rho_{\rm grain} = 3$ g cm^{-3}
$s_{\rm min} = 0.5\ \mu$m
$s_{\rm max} = 5$ cm
Planet properties
$M_{\rm planet} = 15\, M_\oplus$
$a_{\rm planet} = 0.25$ au
$R_{\rm core} \approx 0.00011$ au
Disc properties
$R_{\rm in} = 0.025$ au
$R_{\rm out} = 0.625$ au
$N_{\rm part} \sim 50$ million
$\Sigma = 1700\ {\rm g\ cm^{-3}}\ (R/{\rm au})^{-1}$
-12
-10
-8
-6
log ρ_g [g/cm^3]

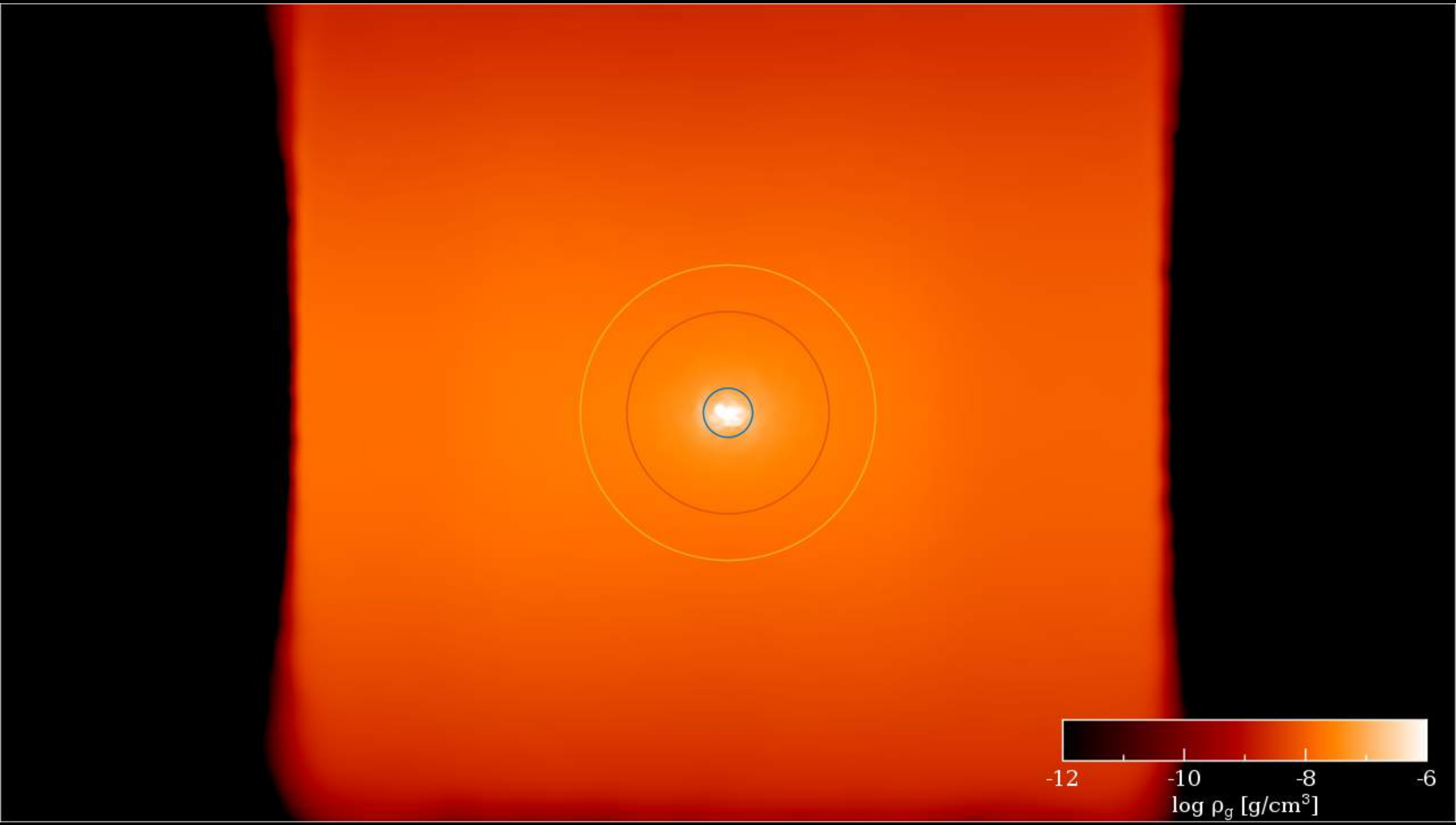
Full disc

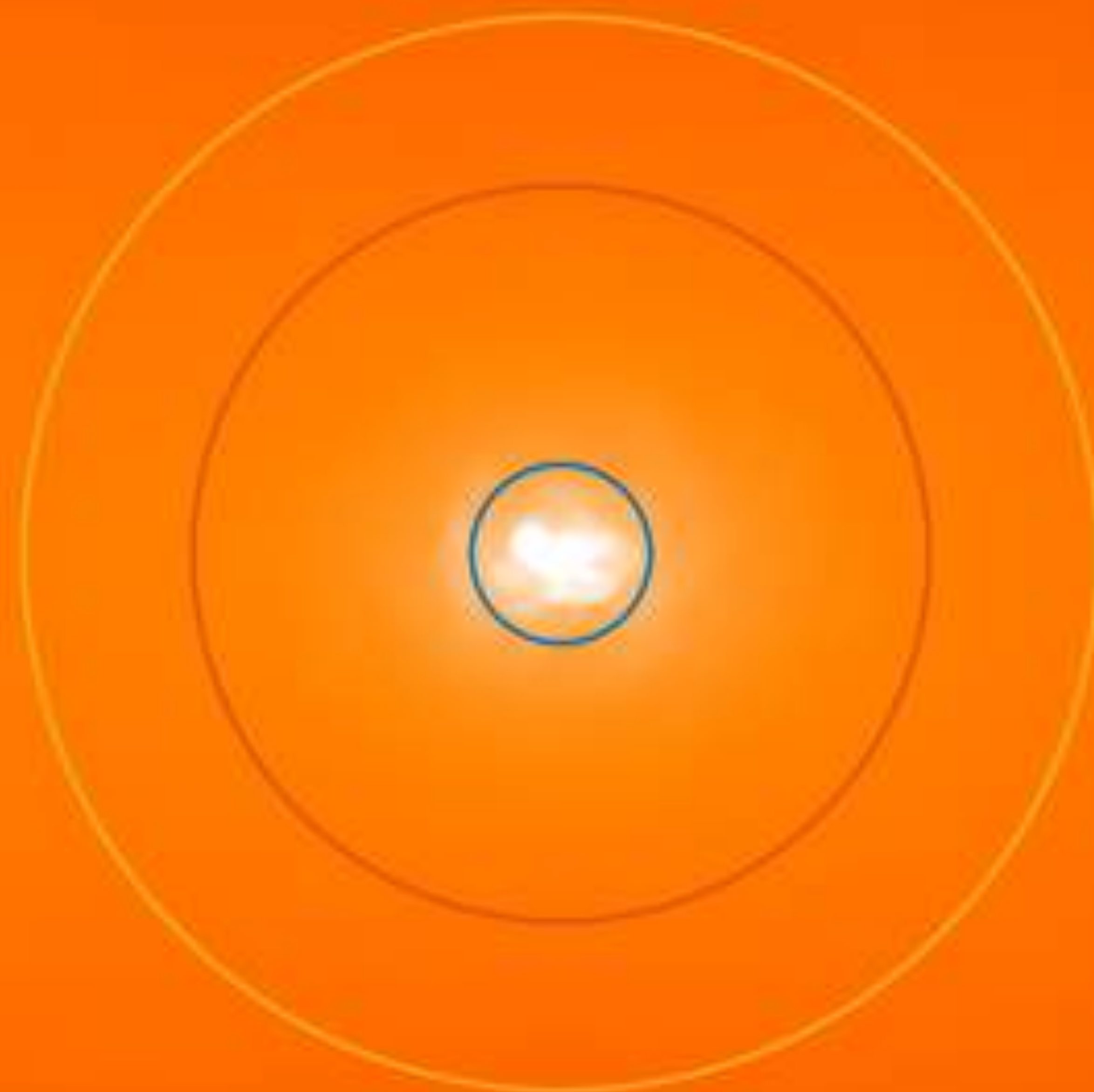


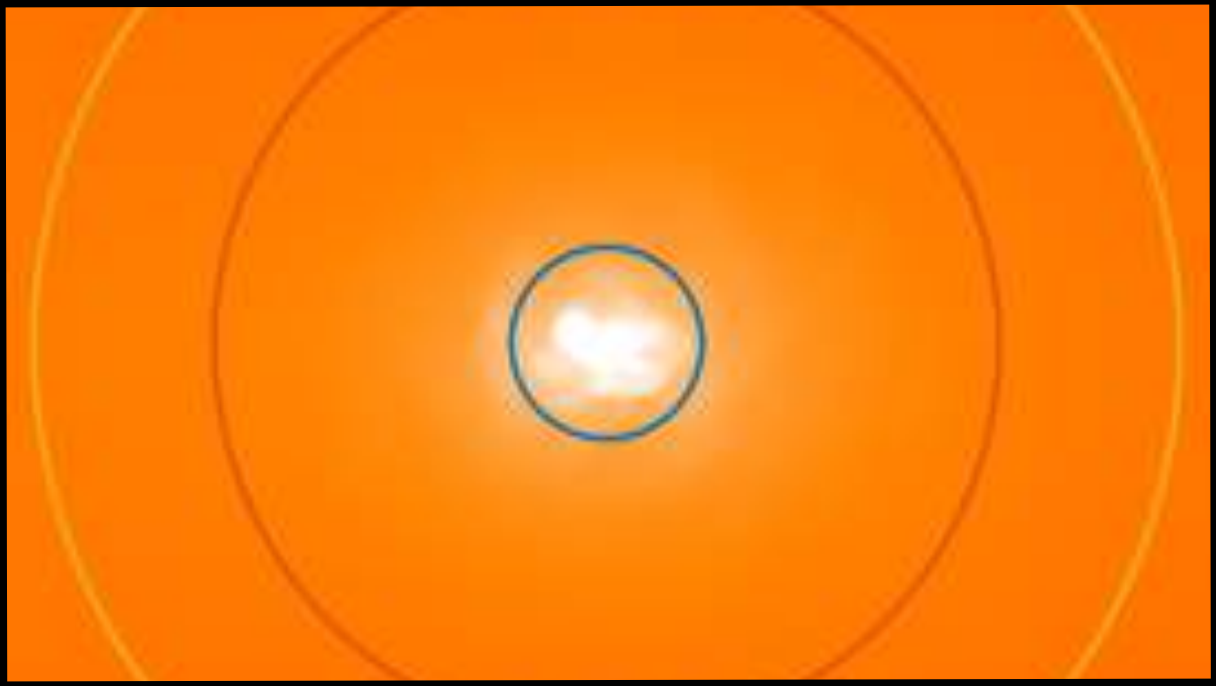
Vertical cross section

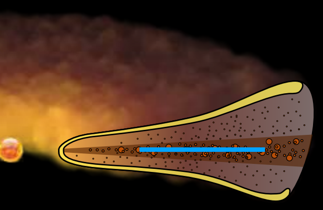
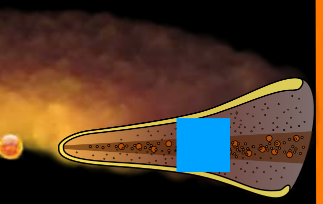
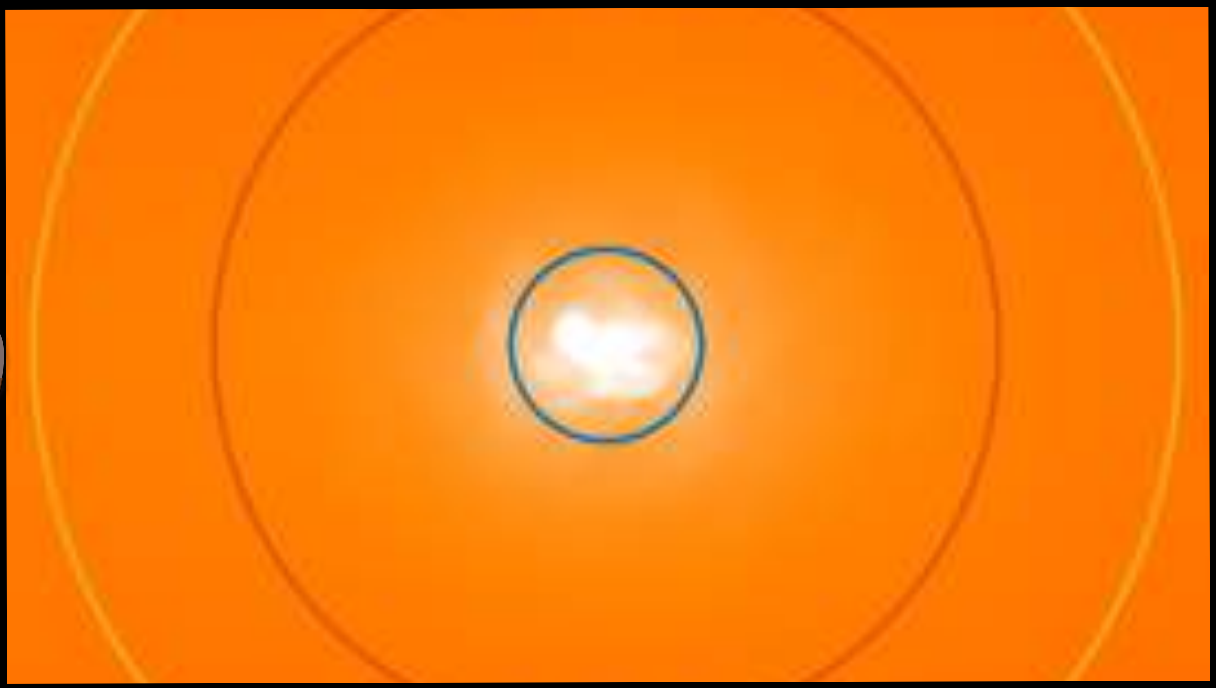
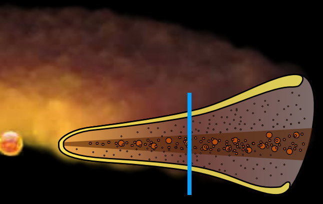


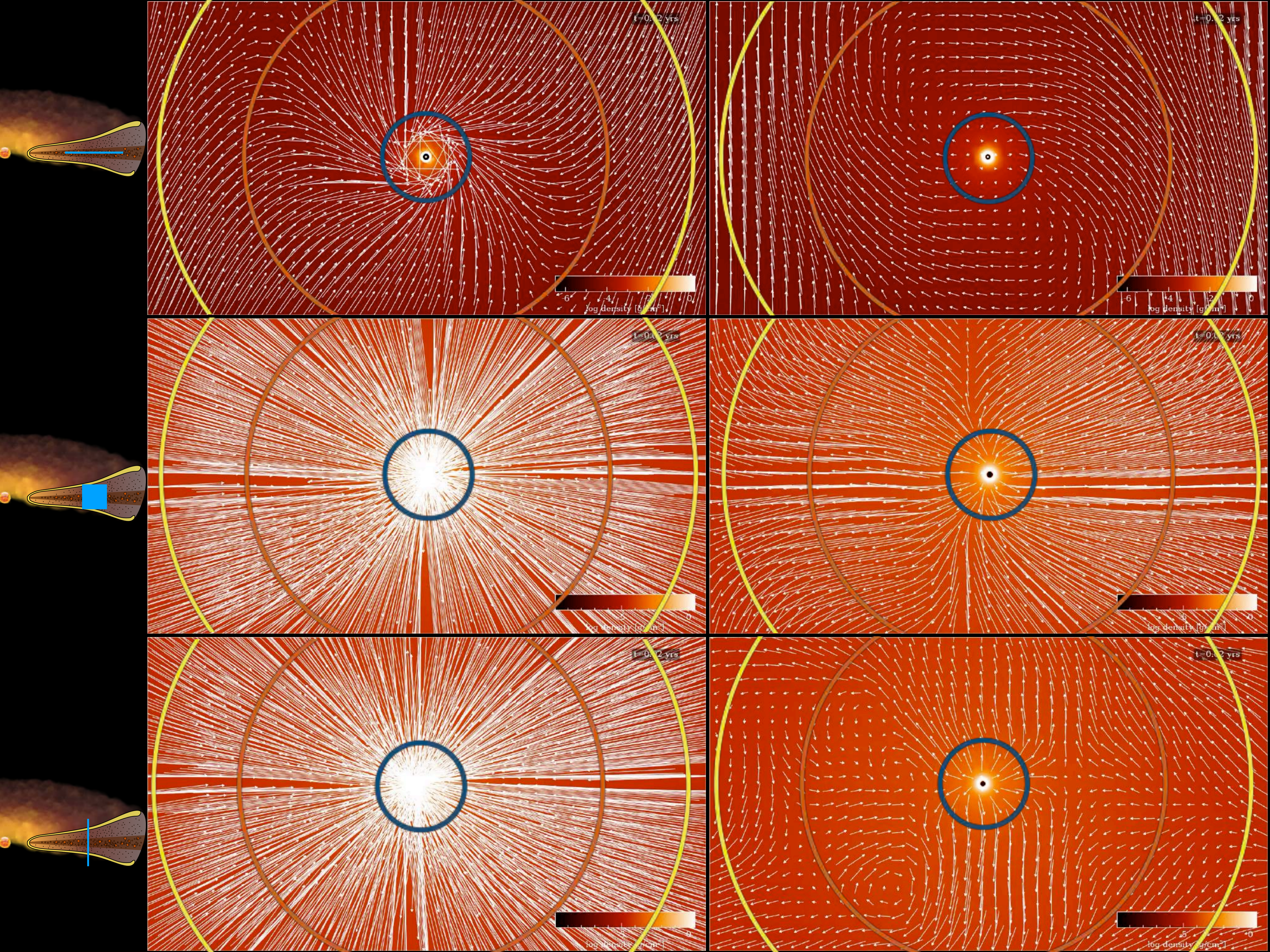
3D cube: rendered

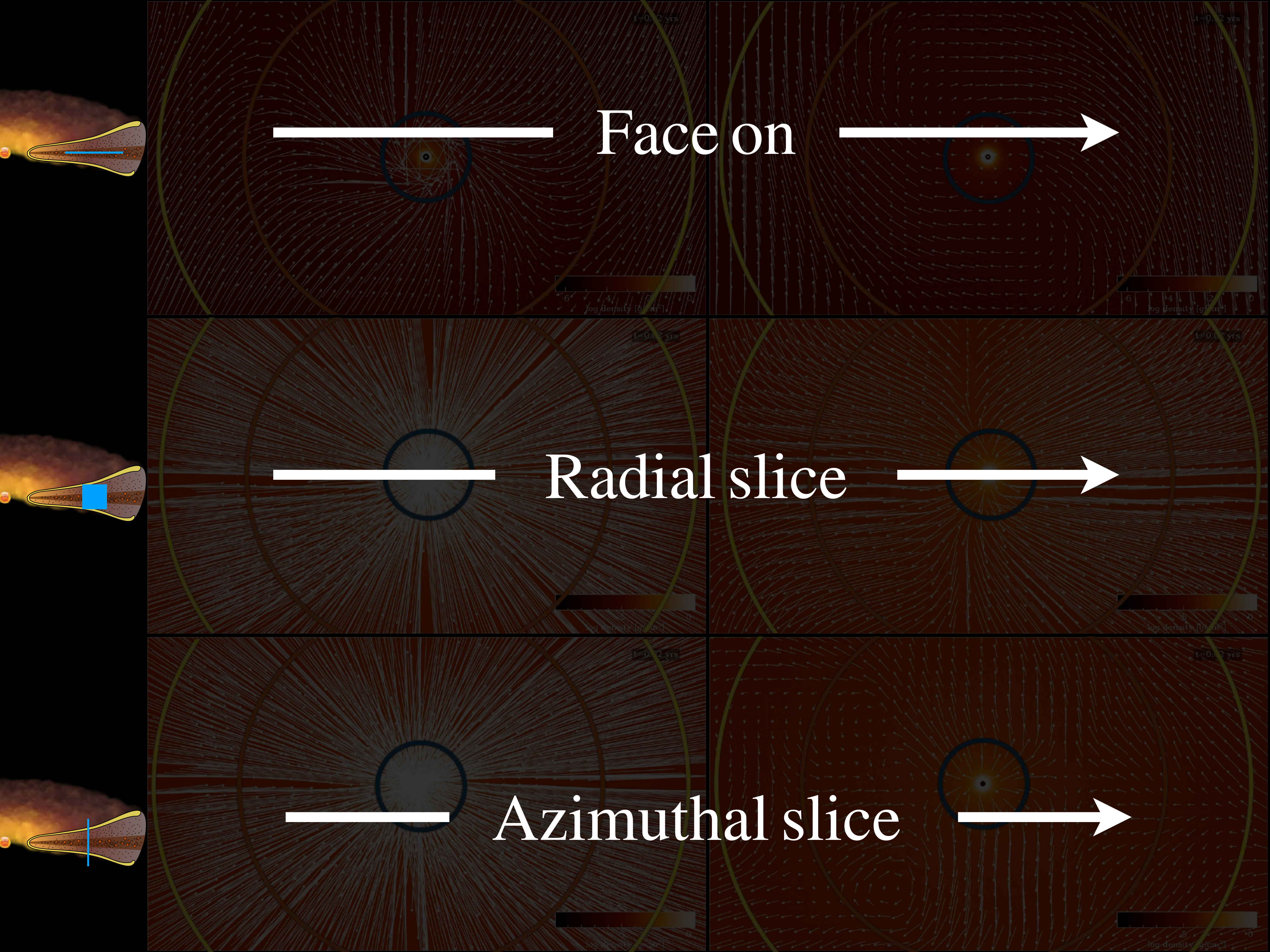

-12
-10
-8
-6
log ρ_g [g/cm^3]

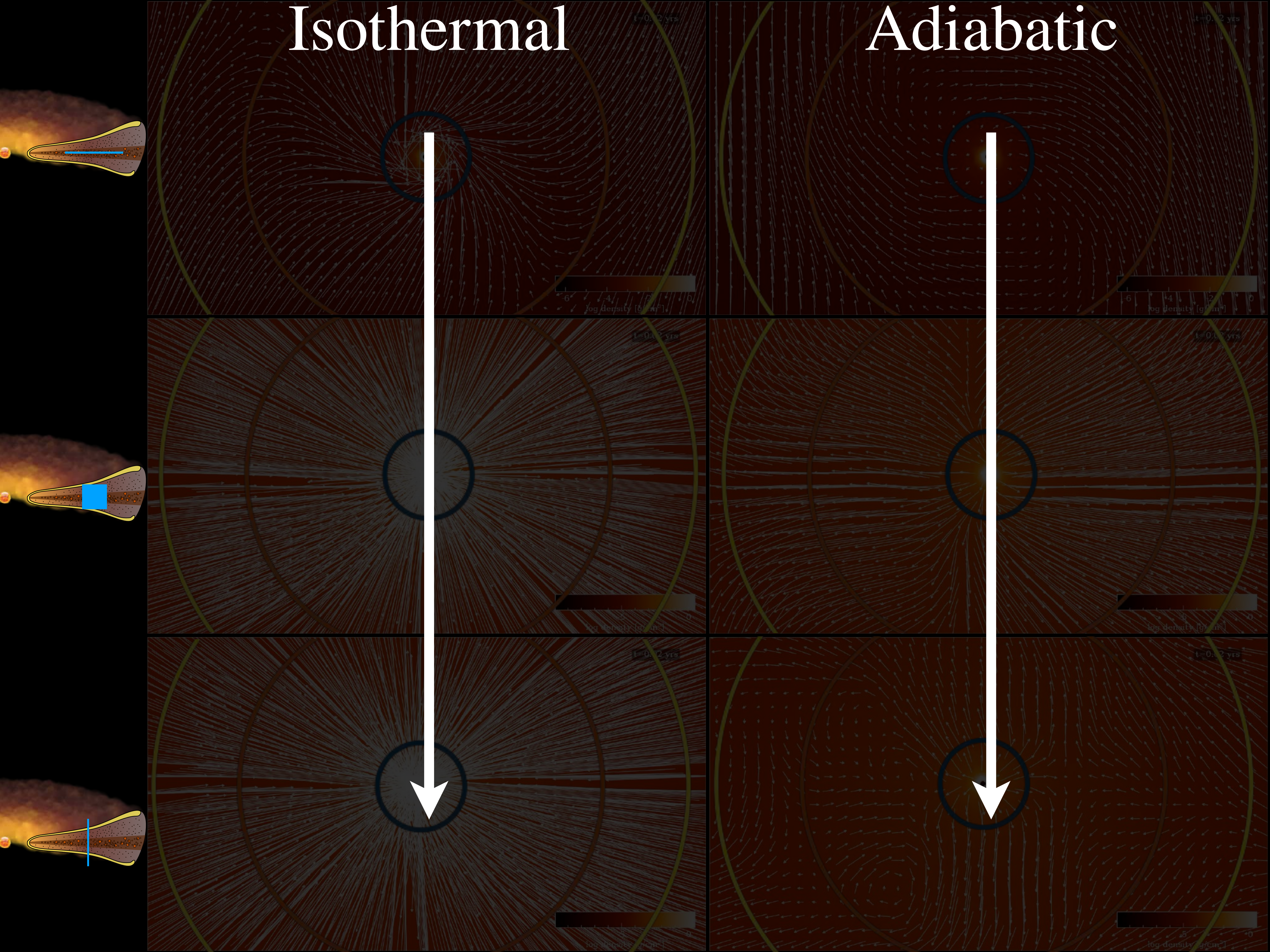


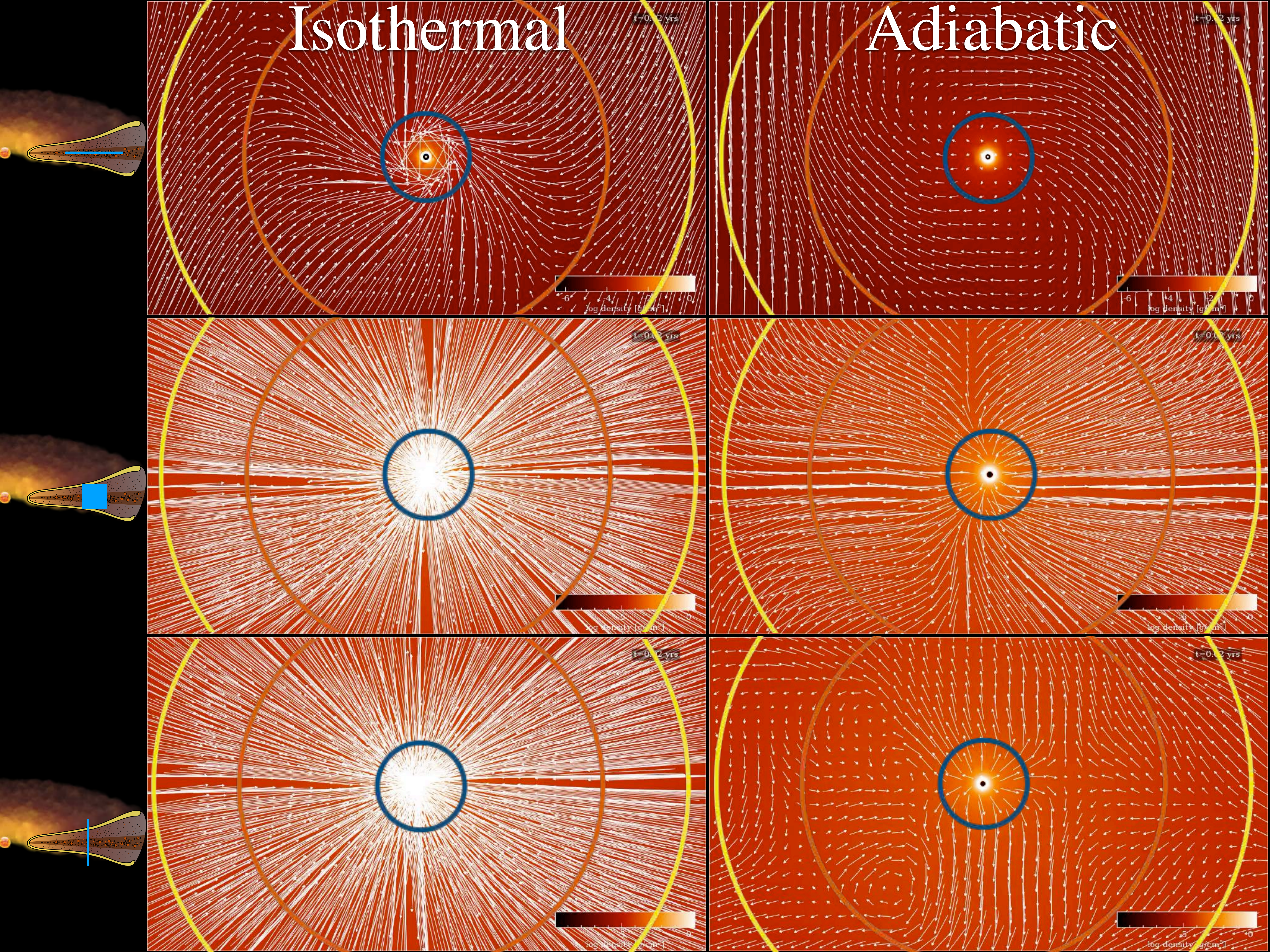


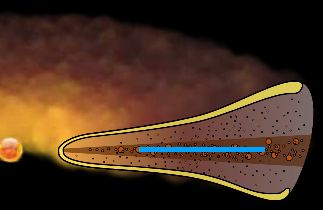
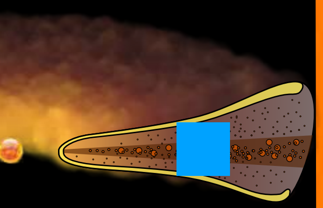
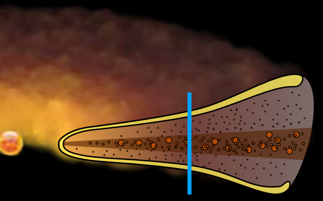
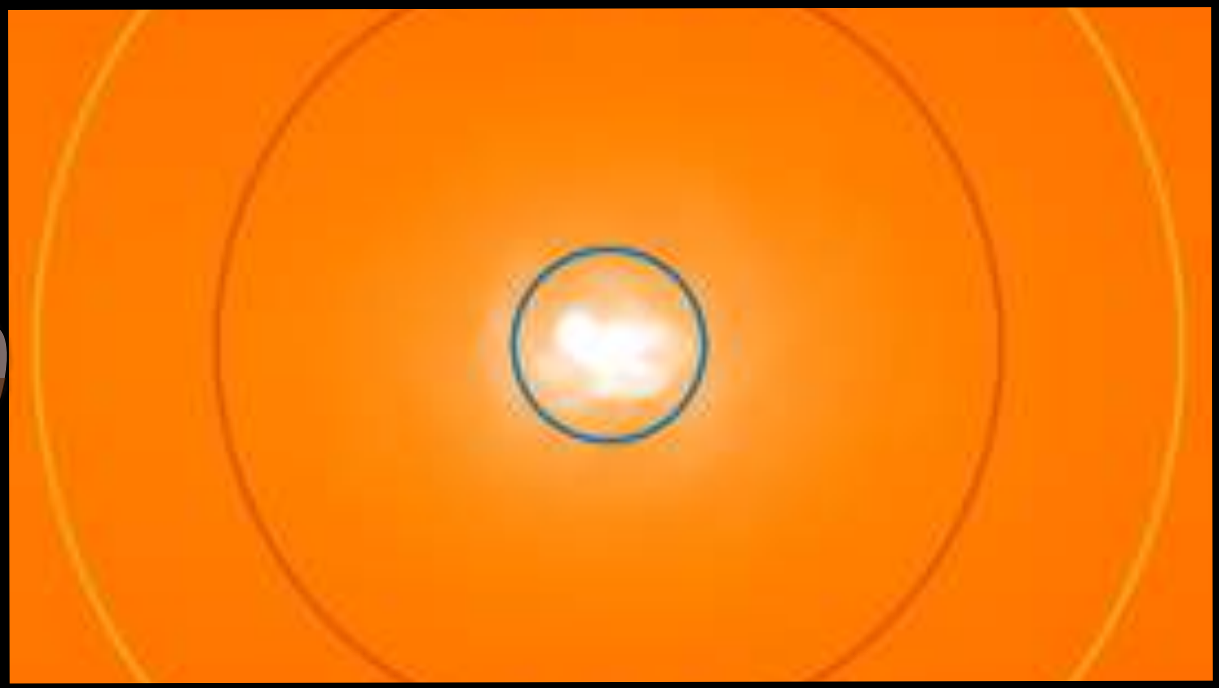





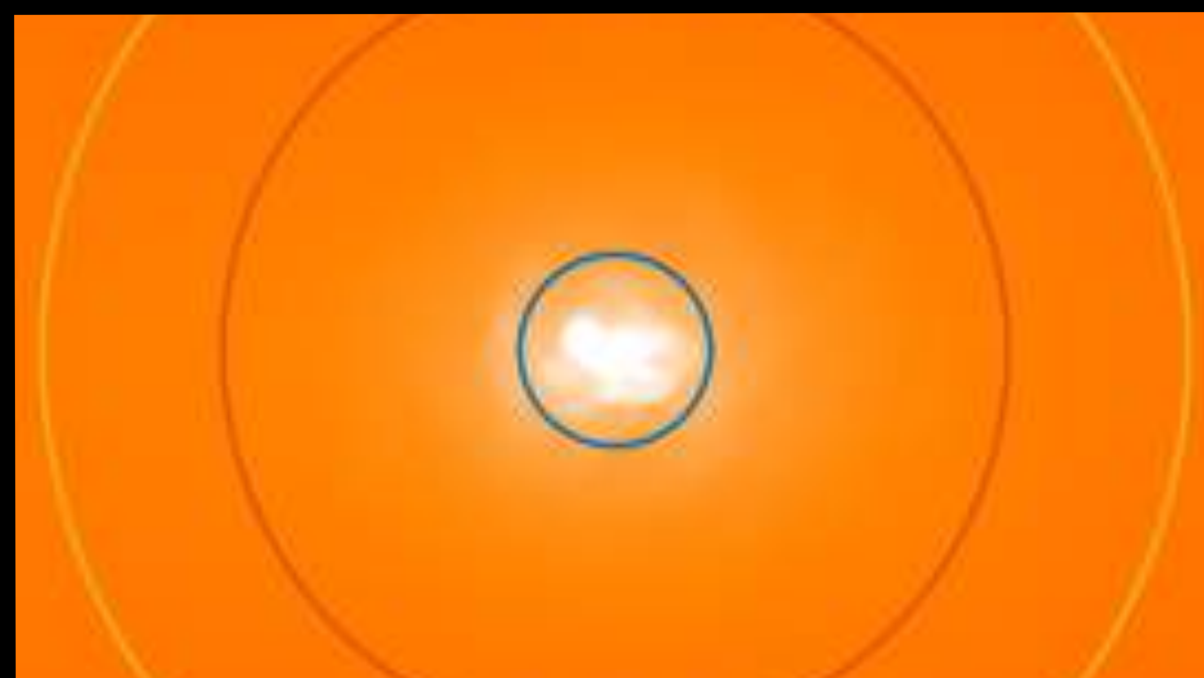

log density [g/cm³]

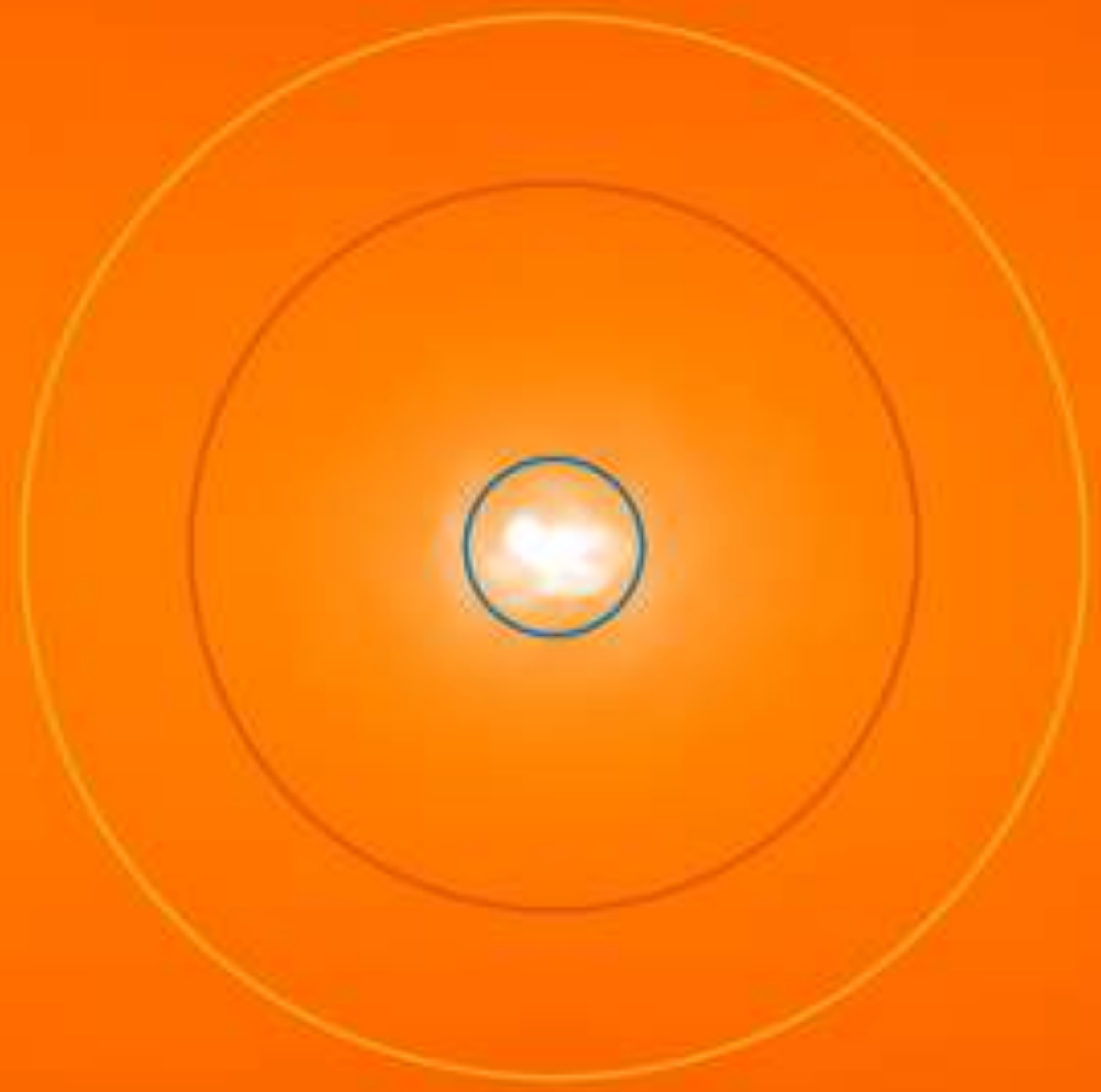

Face on
Radial slice
Azimuthal slice

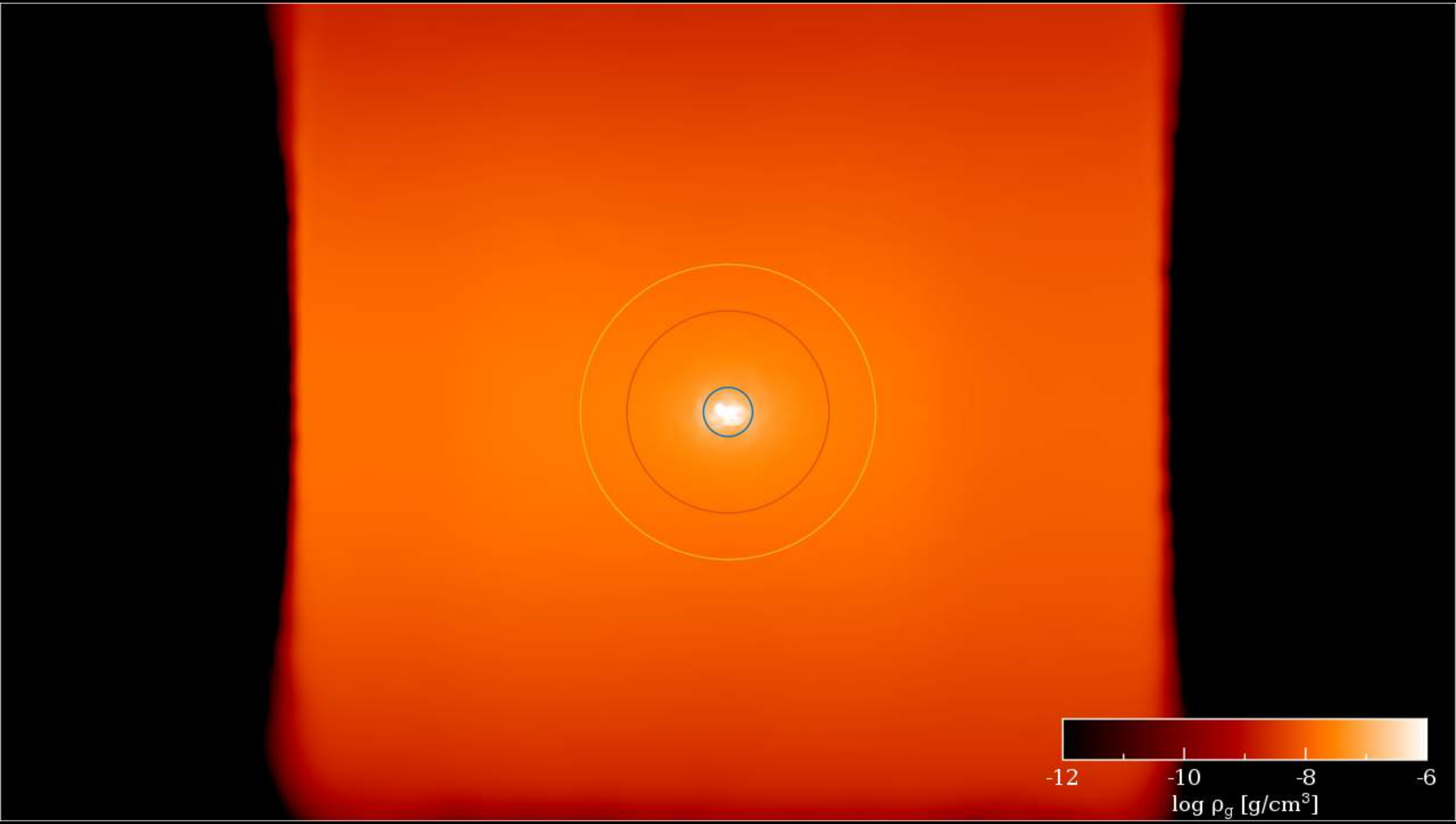

Isothermal
Adiabatic

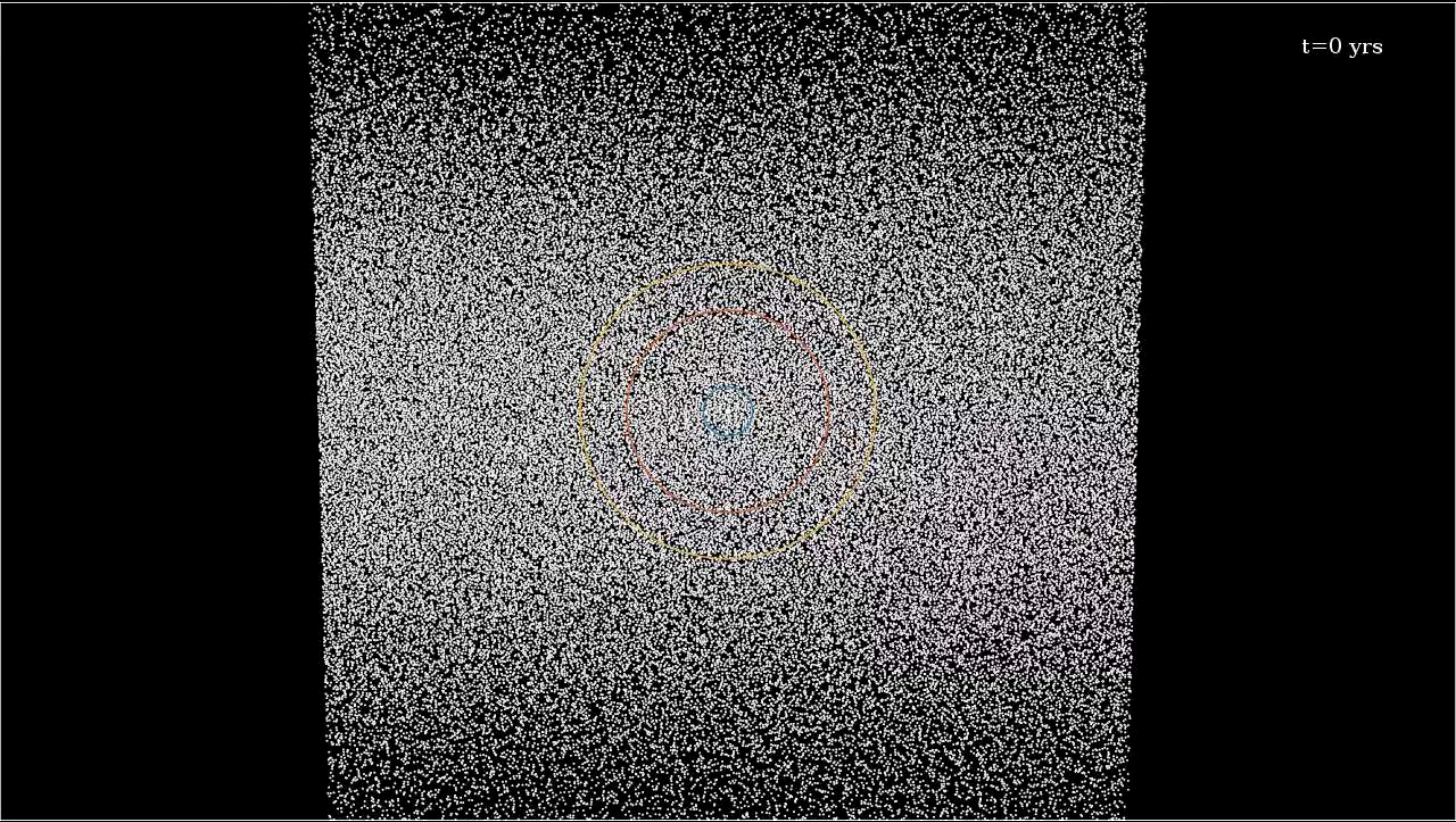

Isothermal
Adiabatic

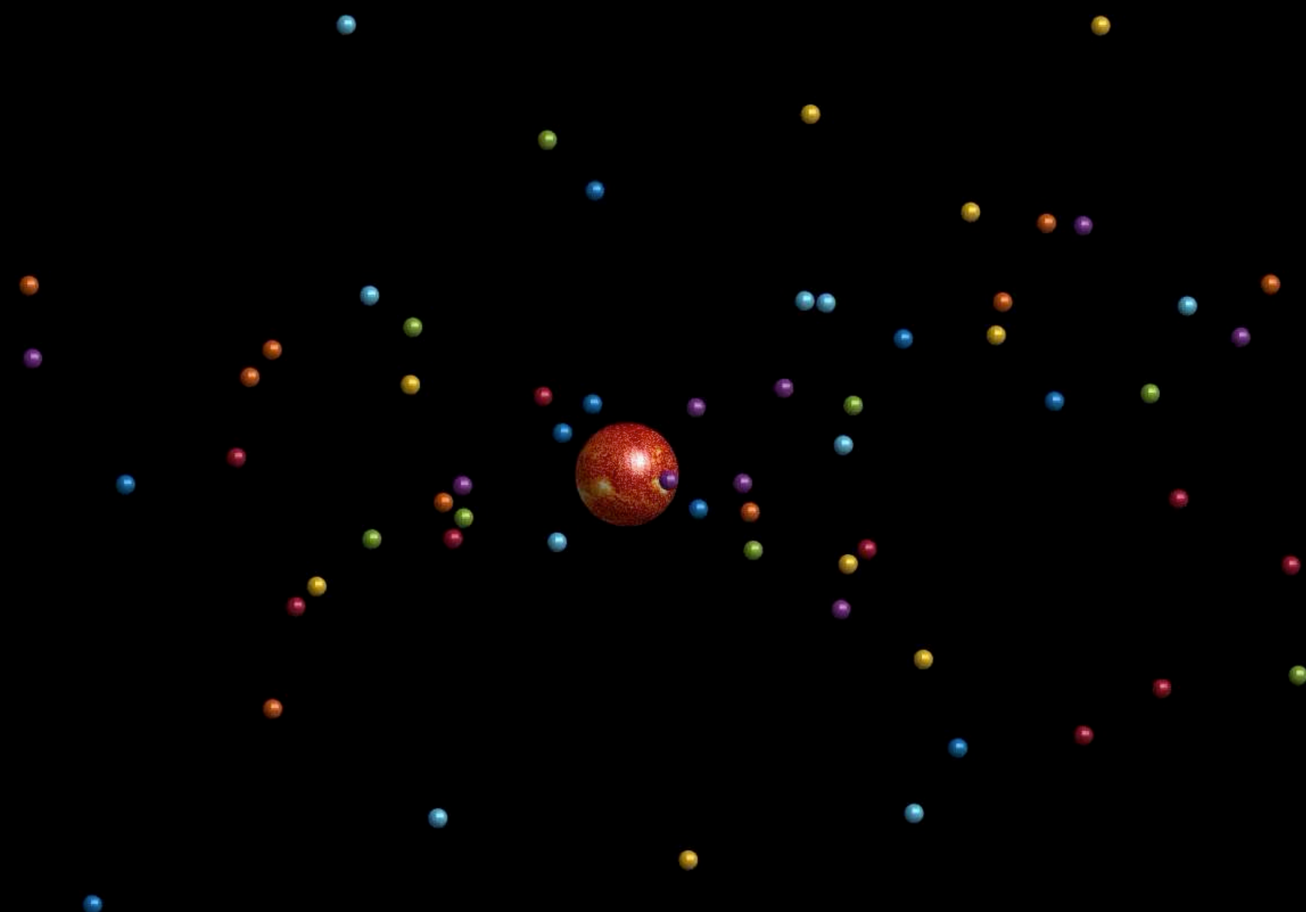


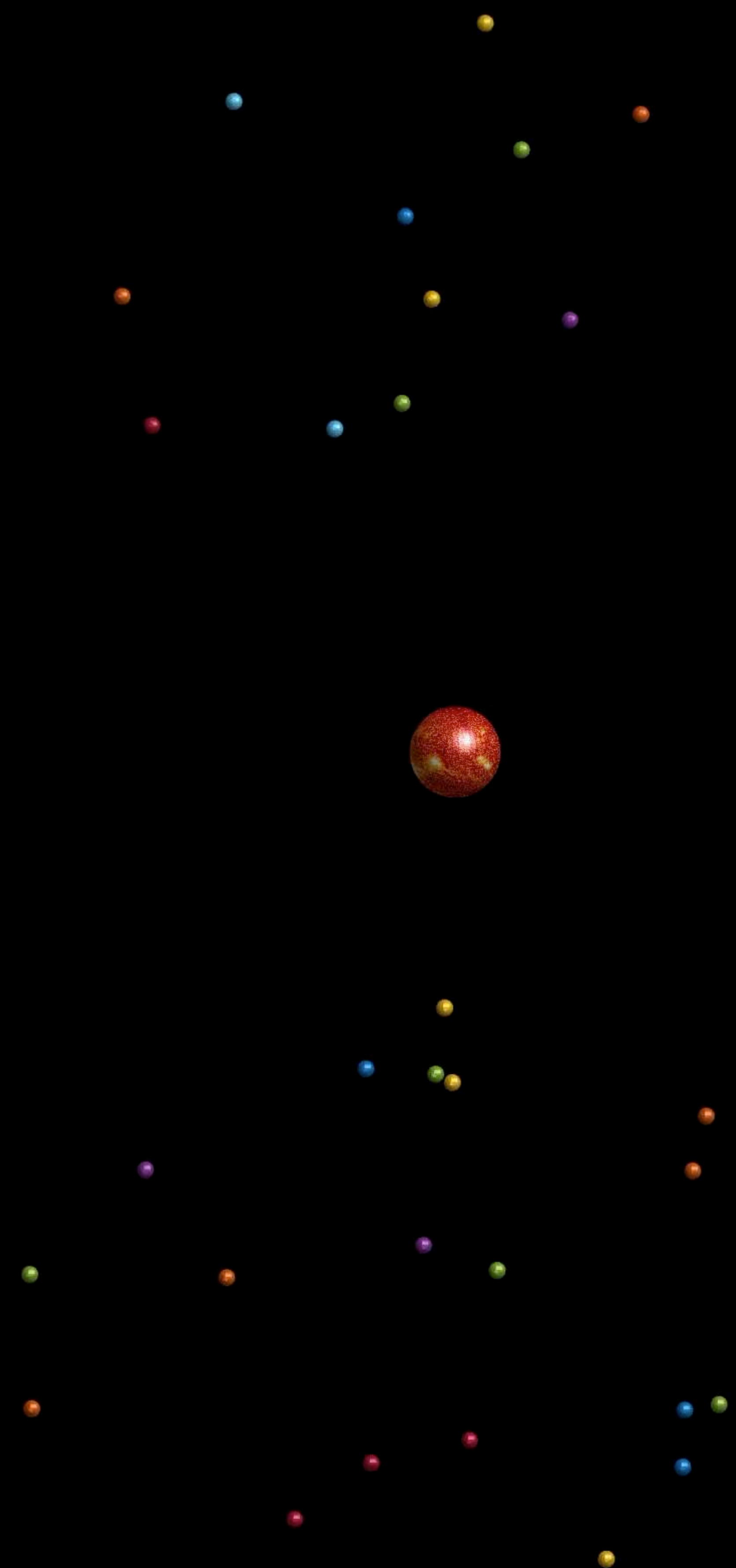


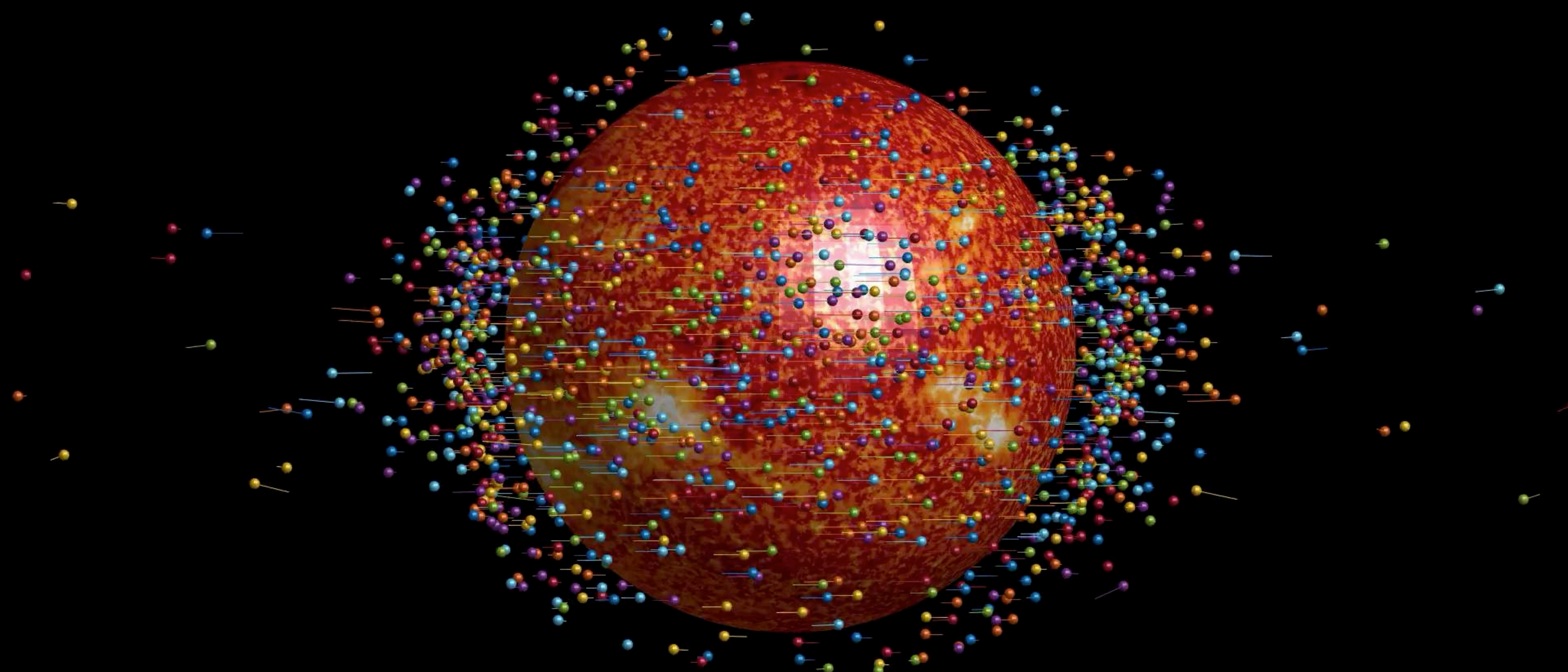

-12
-10
-8
-6
log ρ_g [g/cm^3]



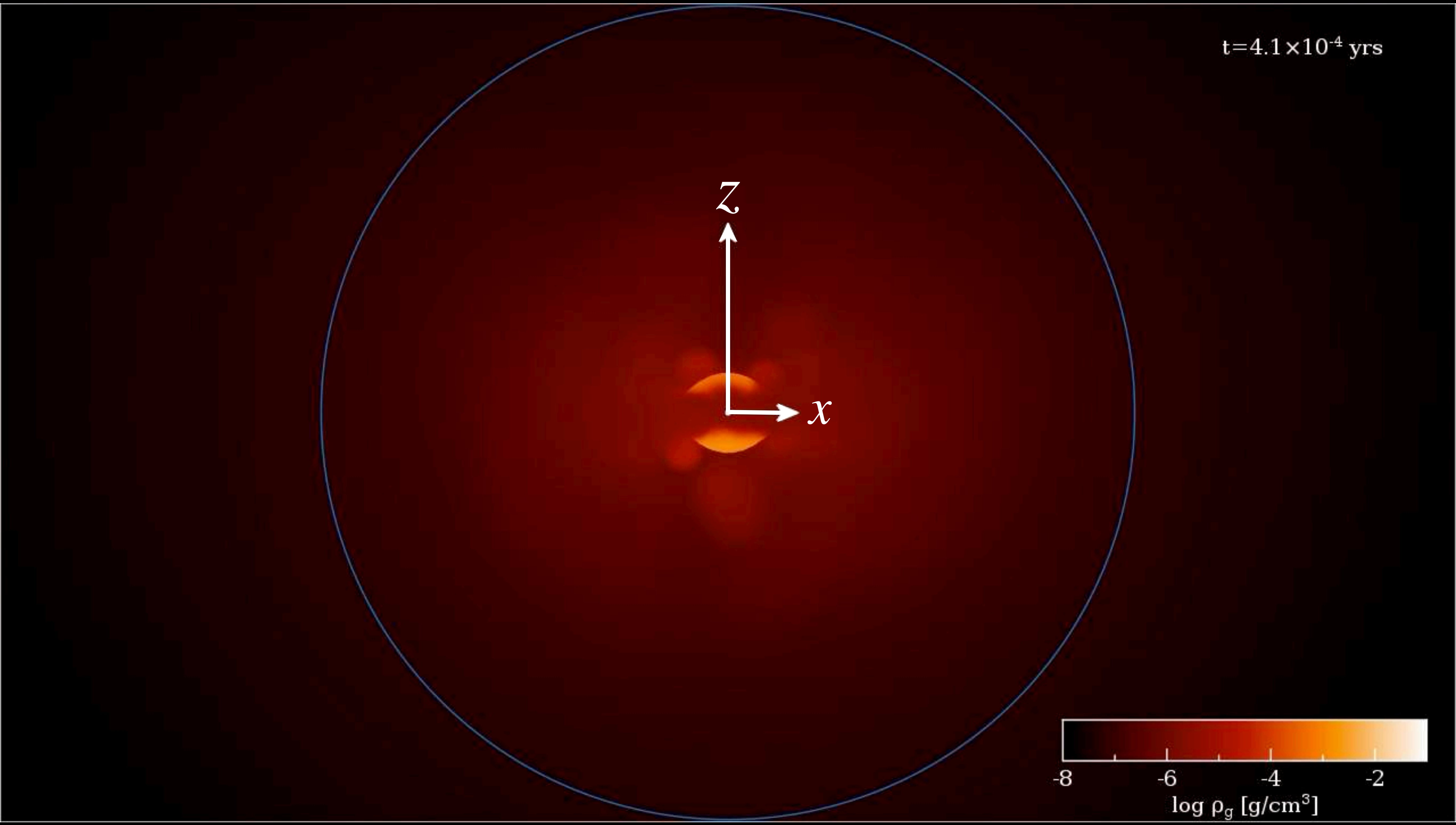
3D cube: particles

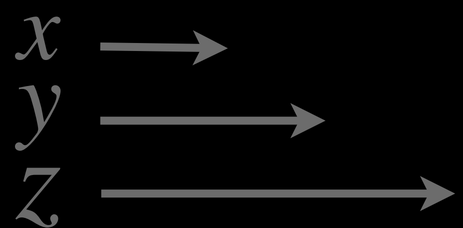




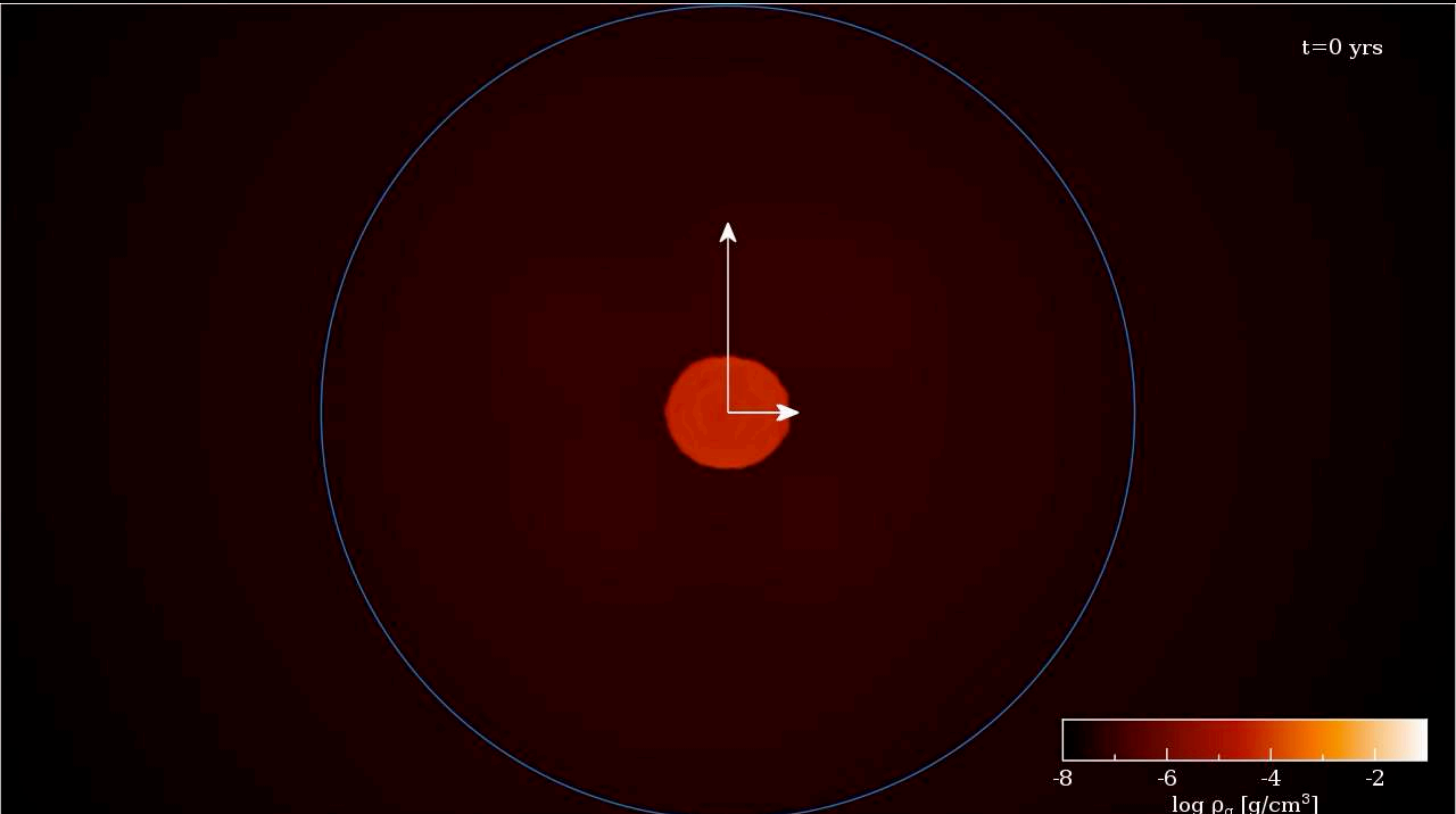


Isothermal





Adiabatic



x →
y →
z →

GAS ACCRETION: RUNAWAY GROWTH

- The hydrostatic atmosphere is described by the structure equations (conservation laws)
- Mass: $$\frac{dM}{dr} = 4\pi r^2 \rho$$
- Momentum (force balance): $$\frac{dP}{dr} = -\frac{GM}{r^2}\rho$$
- Energy: external energy injection (ϵ), volume work (PdV), and internal energy (dU): $$\frac{dL}{dr} = 4\pi r^2 \rho \left(\epsilon - T\frac{\partial S}{\partial t} \right)$$
- Energy transport (radiative vs convective):

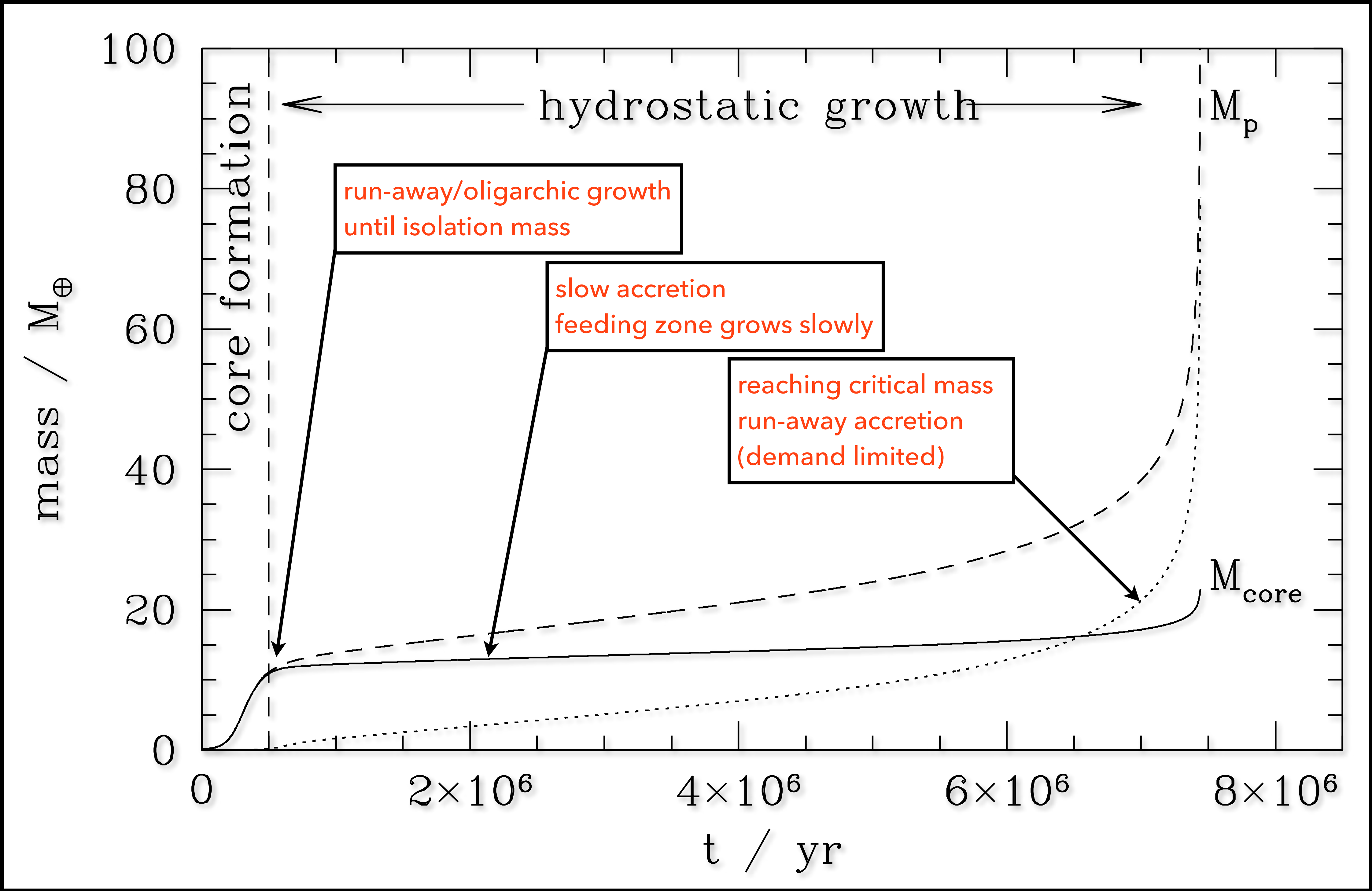
$$\nabla_{\rm rad} = \frac{3}{64\pi\sigma G}\frac{\kappa P L}{T^4 M}$$

$$\frac{dT}{dr} = \frac{T}{P}\frac{dP}{dr}\nabla \quad \text{where} \quad \nabla = \frac{d\ln T}{d\ln P} = \min(\nabla_{\rm rad}, \nabla_{\rm ad})$$

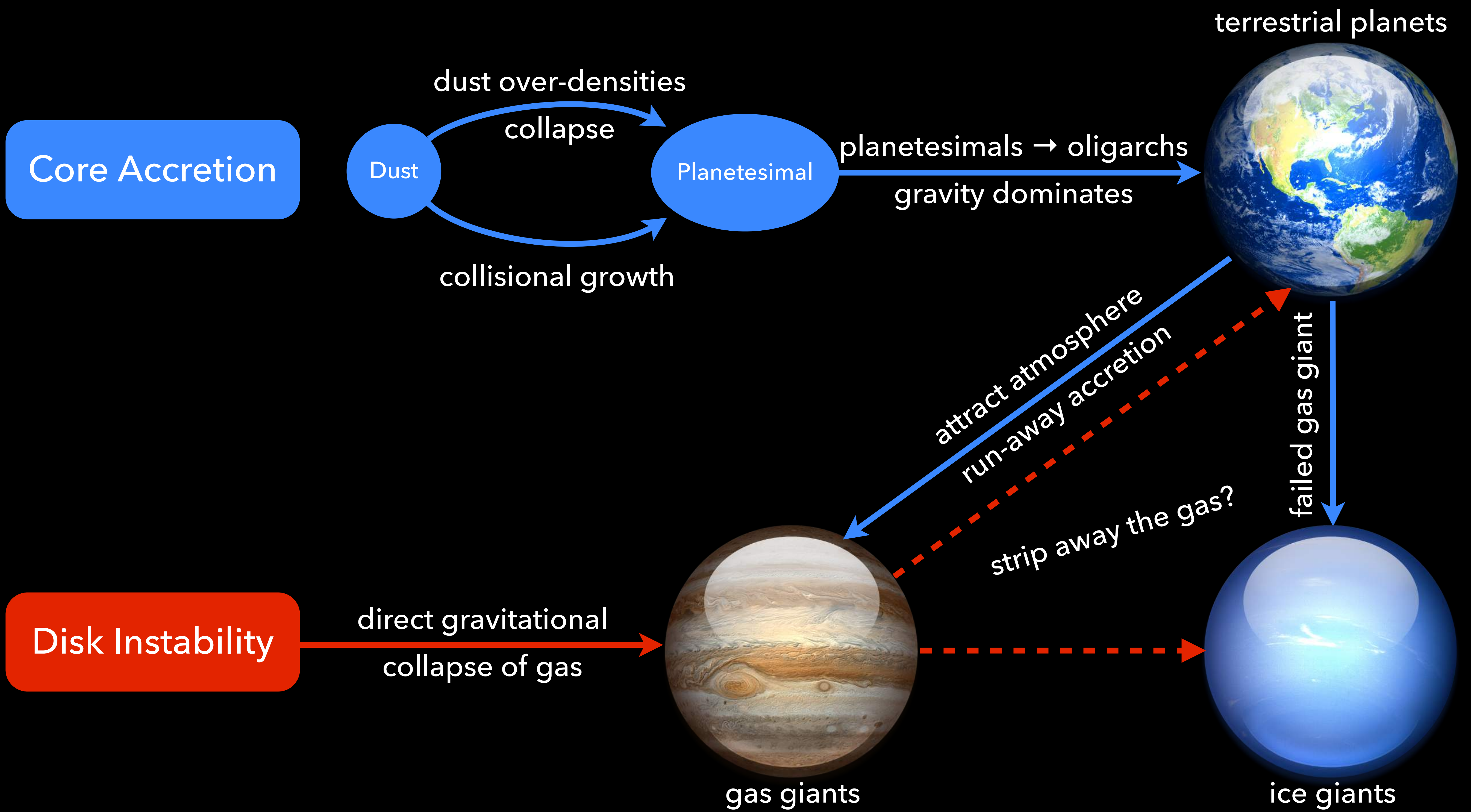
$$\nabla_{\rm ad} = \left.\frac{\partial \ln T}{\partial \ln P}\right|_S$$

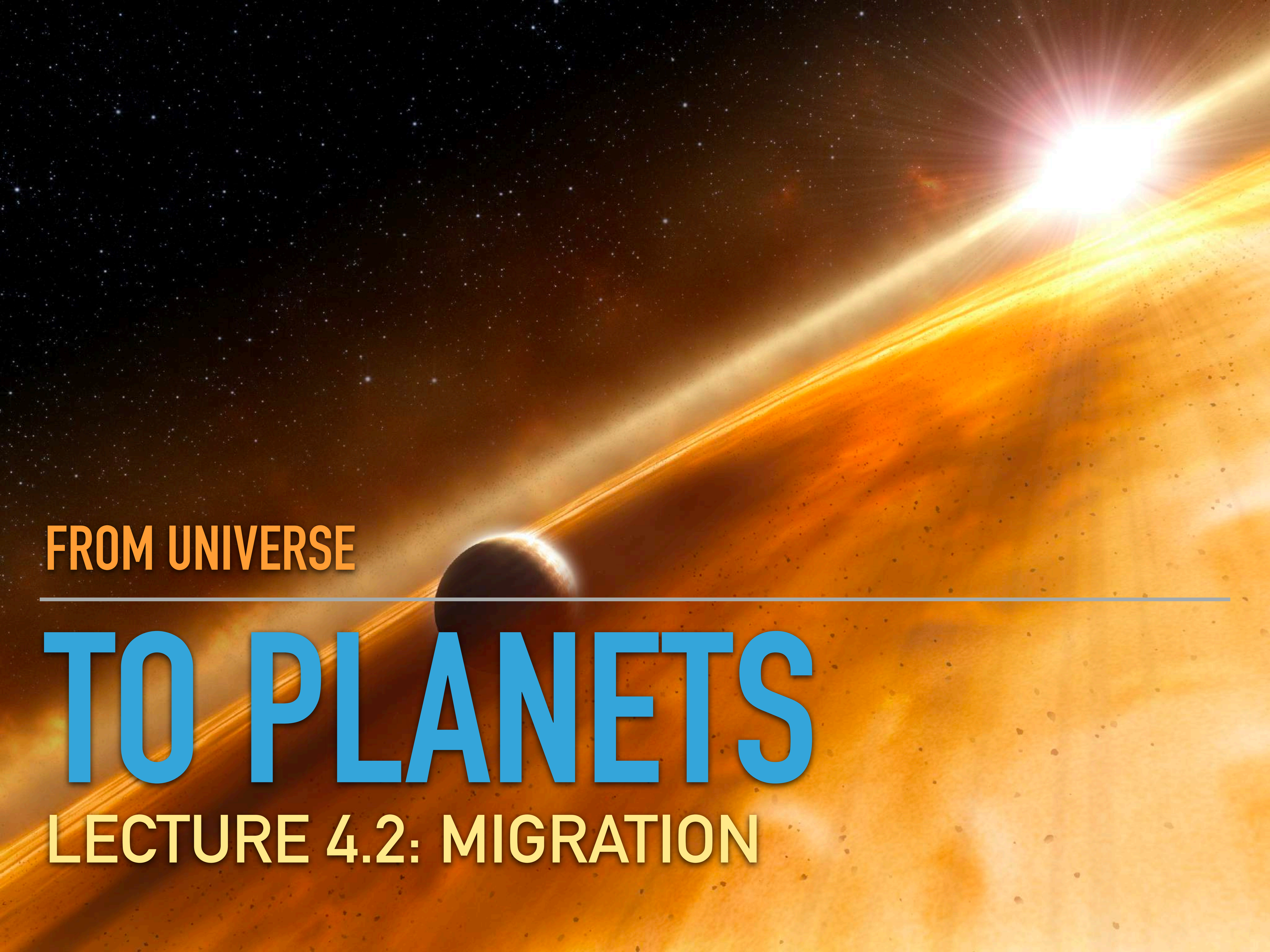
GAS ACCRETION: RUNAWAY GROWTH

- Accounting for boundary conditions (attached vs detached) leads to a step-like growth behaviour:



GAS ACCRETION: RUNAWAY GROWTH





FROM UNIVERSE

TO PLANETS

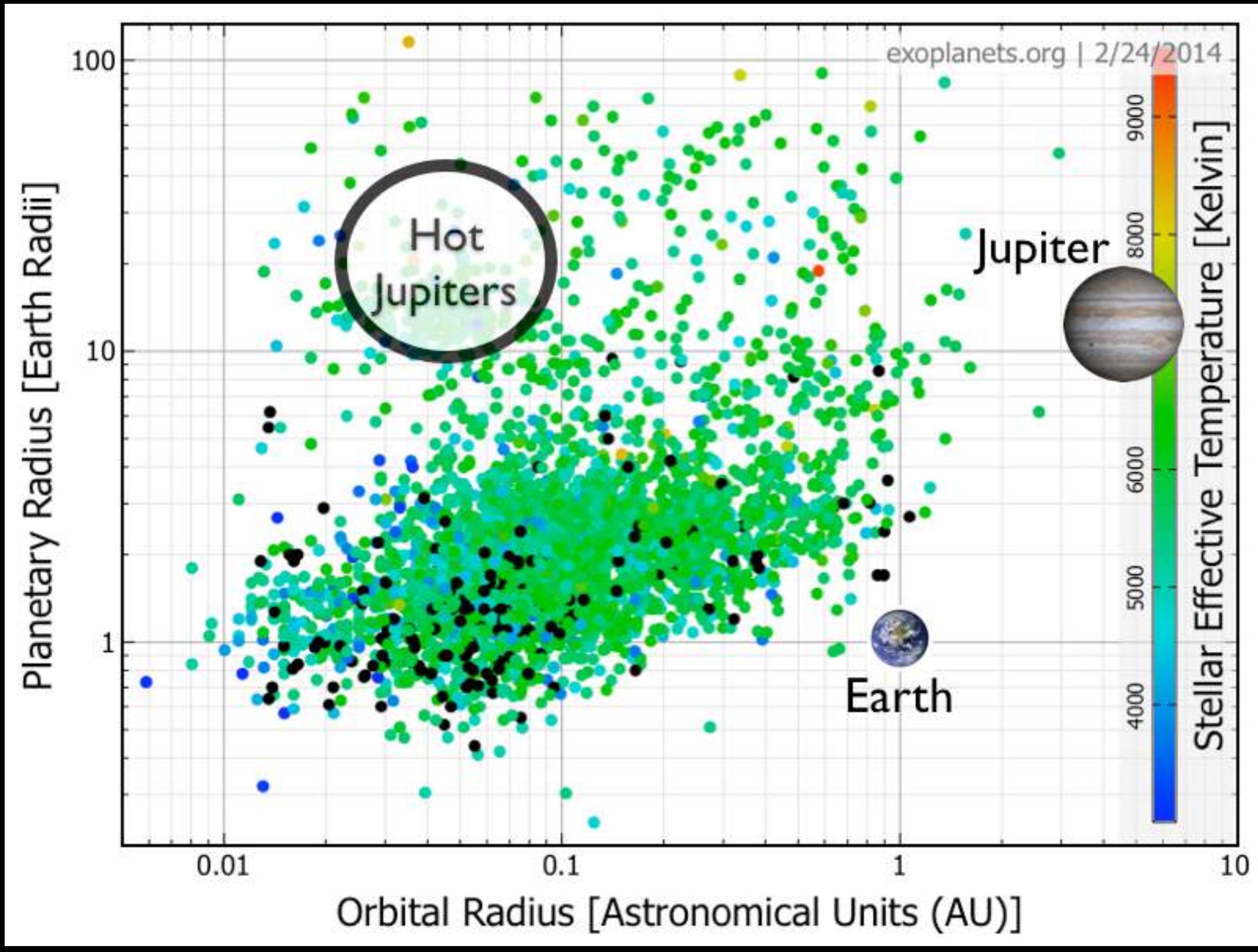
LECTURE 4.2: MIGRATION

MIGRATION

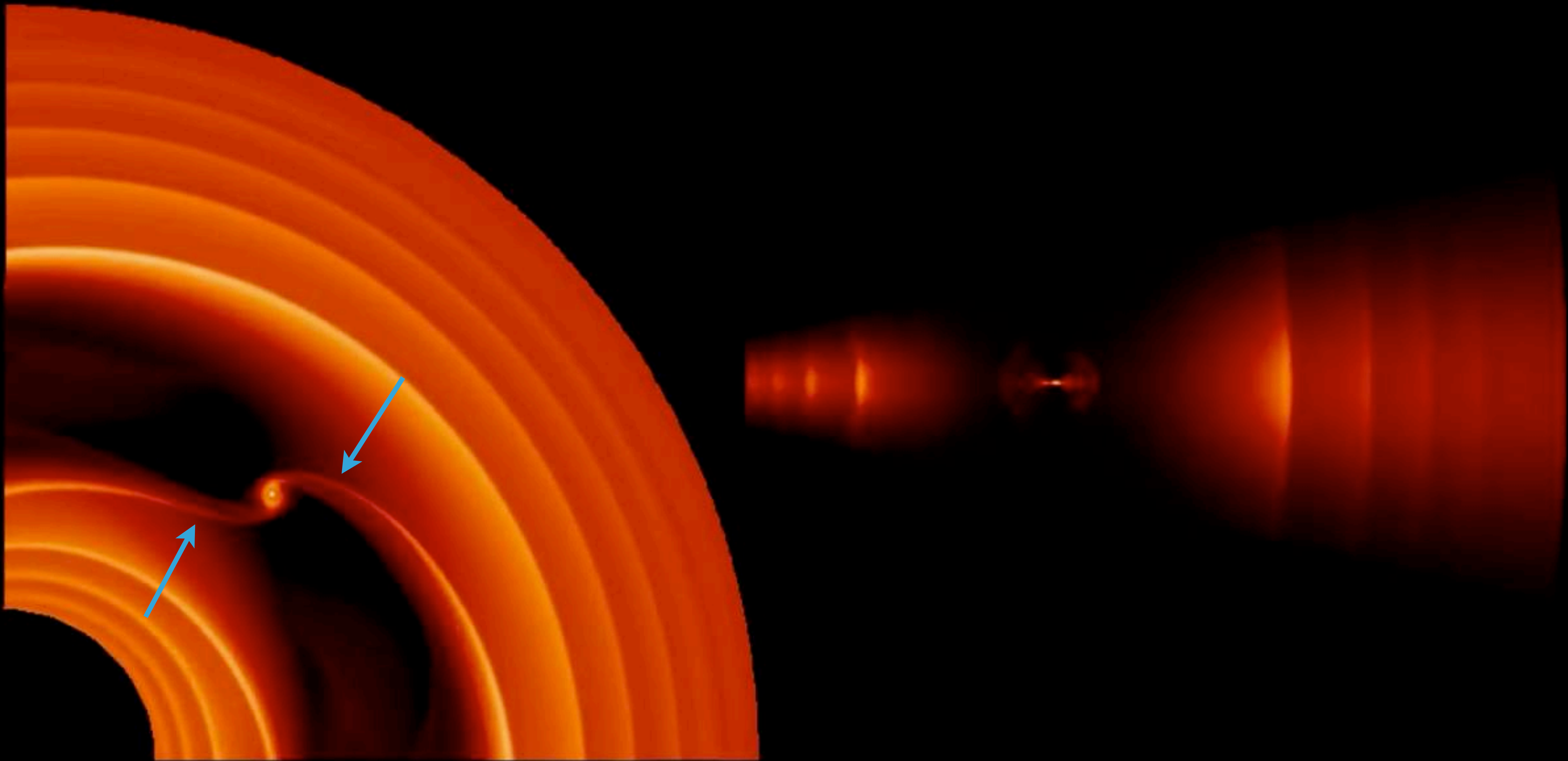
- Disc migration: interactions with the gas disc.
  - Type I: low mass planet.
  - Type II: massive planet in a gap.
  - Type III: turbulent migration (intermediate special case for massive disks).
- Tidal migration: planets whose orbit (or partial orbit) takes it close to the star → causing a bulge on the star.
- Gravitational scattering: interactions with other planets (particularly giant planets) or large number of planetesimals.
- Kozai cycles and tidal friction: planets that are inclined relative to the plane of a binary star (or other planets).

DISC MIGRATION

- Planets create non-axisymmetric, time-dependent gravitational potentials that make density waves in the gas disc (spiral waves).
- These density waves feedback onto the planet through gravitational torques, leading to angular momentum transport and migration.
- Orbital decay due to direct gas drag is negligible at planetary masses.
- Helps explain how hot Jupiters can exist so close to the host star when they preferentially form beyond the snow line.



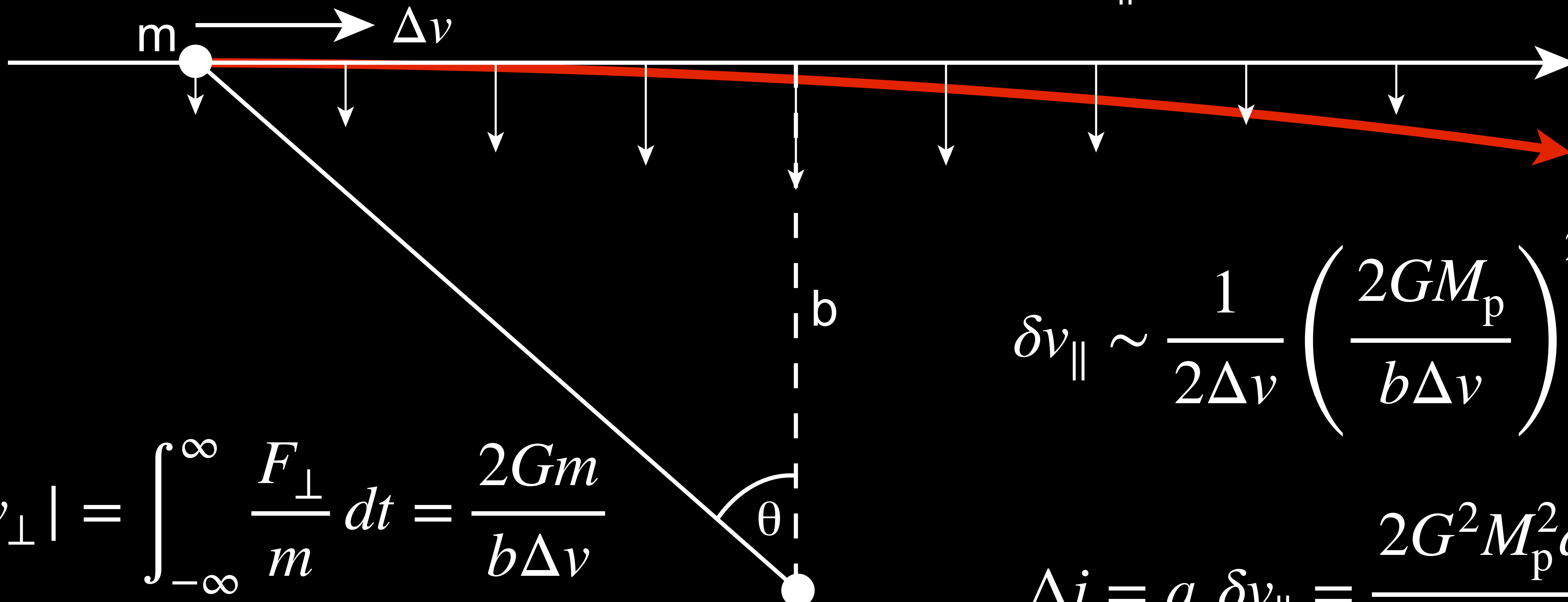
DISC MIGRATION



- The leading density enhancement pulls the planet forward (leading to outward migration), while the trailing density enhancement pulls the planet backwards (leading to inward migration).

DISC MIGRATION: IMPULSE APPROXIMATION

- Assume there is no co-rotating material (Type II migration)
- $\delta v_\perp$ does not change the angular momentum, but conservation of energy implies that a change in $\delta v_\perp$ will alter $\delta v_\parallel$.
- Assume the deflection is small: $[\Delta v^2, \delta v_\parallel^2] \sim 0$



$$\delta v_\parallel \sim \frac{1}{2\Delta v}\left(\frac{2GM_\mathrm{p}}{b\Delta v}\right)^2$$

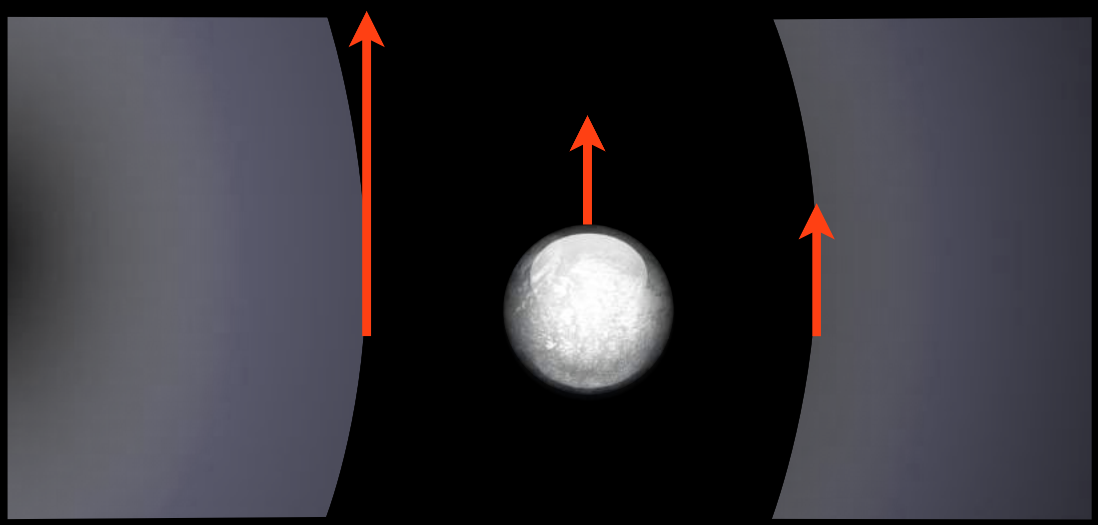
$$|\,\delta v_\perp\,| = \int_{-\infty}^{\infty} \frac{F_\perp}{m}\,dt = \frac{2Gm}{b\Delta v}$$

$$\Delta j = a_\mathrm{p}\delta v_\parallel = \frac{2G^2M_\mathrm{p}^2 a_\mathrm{p}}{b^2\Delta v^3}$$

$$\Delta v^2 = |\,\delta v_\perp\,|^2 + \left(\Delta v - \delta v_\parallel\right)^2$$

DISC MIGRATION: IMPULSE APPROXIMATION

- Gas interior overtakes the planet → net gain in angular momentum. Gas exterior to the planet is overtaken by the planet → net loss.

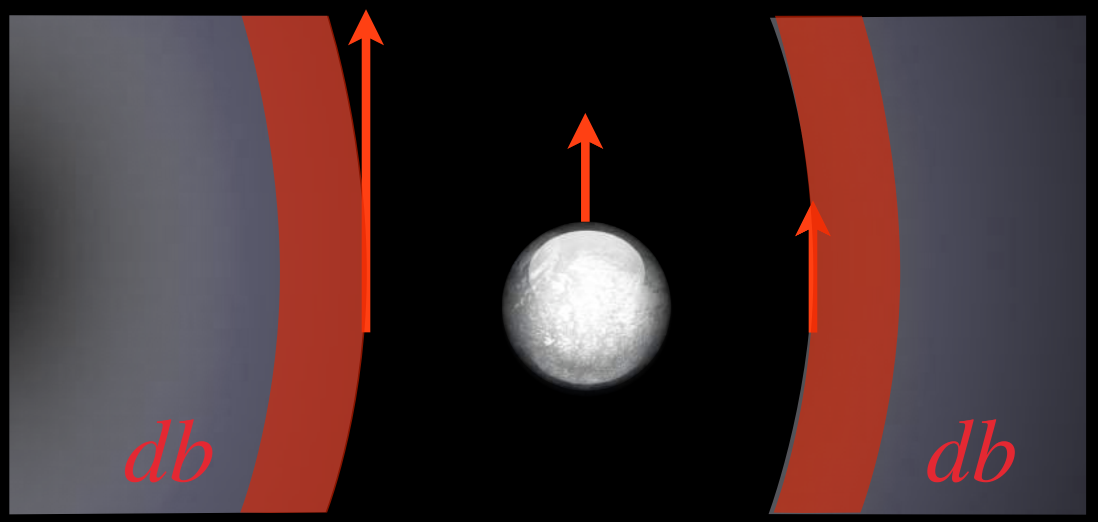


- The interaction is frictional with the net direction of migration depending on the difference between the interior and exterior torques.
- We are assuming linear trajectories (approximately true to circular orbits). Deflections cause departures from this geometry, so we assume disc viscosity is able to restore the geometry by the subsequent pass.

DISC MIGRATION: IMPULSE APPROXIMATION

- Net torque requires integrating over gas within small annuli on either side of the planet (i.e. from b to $b + db$) with a mass:

$$dm \approx 2\pi a_p \Sigma db$$



- Differences in orbital frequencies:

$$\Delta t = \frac{2\pi}{|\Omega - \Omega_p|}$$

- For small displacements ($b \ll a_p$) a first order expansion of the frequencies gives:

$$|\Omega - \Omega_p| \approx \left| \frac{d\Omega_K}{da} \right| b = \frac{3\Omega_K}{2a_p} b$$

- We get the total temporal change in angular momentum by integrating over the exchange between all interacting gas parcels per unit time:

$$\frac{dJ}{dt} = -\int \frac{\Delta j \, dm}{\Delta t}$$

DISC MIGRATION: IMPULSE APPROXIMATION

- Assuming $\Delta v \approx a_\mathrm{p} |\Omega - \Omega_\mathrm{p}| \approx (3/2)\Omega_\mathrm{p} b$ and constant surface density, we can then integrate:

$$\frac{dJ}{dt} = -\int_{b_\mathrm{min}}^{\infty} \frac{8G^2 M_\mathrm{p}^2 \Sigma a_\mathrm{p}}{9\Omega_\mathrm{K}^2 b^4}\, db = -\frac{8G^2 M_\mathrm{p}^2 \Sigma a_\mathrm{p}}{27\Omega_\mathrm{K}^2 b_\mathrm{min}^3}$$

- The migration time scale varies as: (massive planets migrate faster)

$$\tau_\mathrm{mig} = \frac{J}{dJ/dt} \propto \frac{1}{M_\mathrm{P}}$$

- More rigorous derivations require looking at resonances:
  - Co-rotation: $\Omega = \Omega_\mathrm{p}$
    (if we neglect gas pressure and self gravity)
  - Lindblad: $m(\Omega - \Omega_\mathrm{p}) = \pm\kappa$ where $m = 1, 2, 3, \ldots$ and κ is the epicyclic frequency (frequency at which a radially displaced fluid parcel will oscillate):

$$r_\mathrm{LR} = a_\mathrm{p}\left(\frac{m \pm 1}{m}\right)^{2/3}$$

DISC MIGRATION: TYPE I

- After a *loooong* derivation of the torques, the total torque can be derived by a summation of all resonances:

$$\text{(resonance strength)} \times \text{(gas mass at resonance)}$$

$$\Gamma = \sum_{m=1}^{\infty} \Gamma^m_{\rm OLR} + \sum_{m=1}^{\infty} \Gamma^m_{\rm ILR} + \Gamma_{\rm CR}$$

- Skipping to the end…

$$\Gamma_{\rm tot} = -(1.36 + 0.54\mathrm{p})\left(\frac{M_{\rm p}}{M_*}\right)^2 \frac{\Sigma a_{\rm p}^4 \Omega_{\rm K}^2}{h^2} \quad \text{where} \quad \Sigma \propto \left(R/R_0\right)^{-p}$$

- Which gives a migration timescale of:

$$\tau_{\rm I} = (2.7 + 1.1\mathrm{p})^{-1} \frac{M_{\rm p}}{M_*} \frac{M_*}{\Sigma a_{\rm p}^2 \Omega_{\rm K}} \propto \frac{1}{M_{\rm p}}$$

- The same scaling as our simple derivation.

DISC MIGRATION: GAP OPENING

- Recall the outer disc extracts angular momentum (moves out) and the inner disc loses it (moves in) → gap.
- Two conditions that need to be met for a gap opening:
  - Thermal: $r_{\rm H} \gtrsim H$ (prevents accretion above/below the mid-plane). This implies a mass ratio:

$$q = \frac{M_{\rm p}}{M_*} \geq 3h^3 \qquad \left(\text{typically}: M_{\rm p} \gtrsim 0.13 M_{\rm Jupiter}\right)$$

  - Viscous: $\tau_{\rm close} \gtrsim \tau_{\rm open}$ (viscosity refills the gap; usually the criterion controlling gap opening). Assuming that $b_{\rm min} = r_{\rm H}$ and $\nu = \alpha c_{\rm s} H$: $\left(\text{typically}: M_{\rm p} \gtrsim 2.5 M_{\rm Jupiter}\right)$

$$\left(\frac{dJ}{dt}\right)_{\rm LR} = -\frac{8G^2M_{\rm p}^2\Sigma a_{\rm p}}{27\Omega_{\rm K}^2 b_{\rm min}^3} \geq 3\pi\nu\Sigma a_{\rm p}^2\Omega_{\rm K} = \left(\frac{dJ}{dt}\right)_{\rm visc} \longrightarrow \quad q \geq \frac{243\pi}{8}\alpha h^2$$

DISC MIGRATION: TYPE II

- Once the gap is opened, the gas is being pushed away from the planet and the torques diminish (slows down).
- The planet is kept in the middle of the gap and migration happens on viscous timescales:

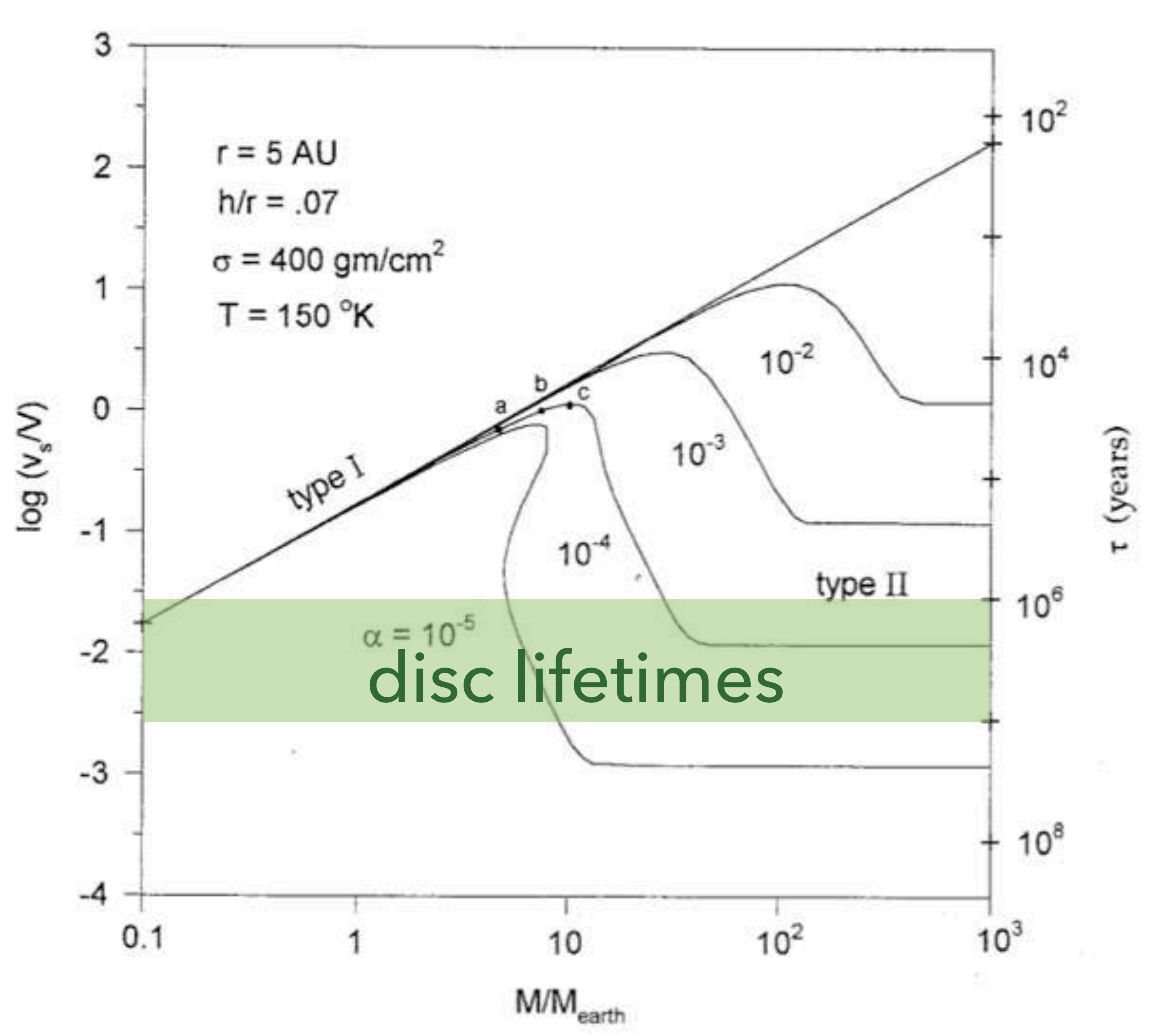
$$\tau_{\rm II} = \frac{a_{\rm p}^2}{\nu} = \frac{1}{\alpha} h^2 \Omega_{\rm K}^{-1}$$

- Only depends on the mass of the star and characteristics of the disc (independent of $M_{\rm p}$). However, this is only valid if the planet is not too massive. Two regimes:
  - Disc dominated
  - Planet dominated

$$\tau_{\rm II} = \begin{cases} \tau_{\rm visc}, & \text{if } B \gg 1 \\ \tau_{\rm visc} B, & \text{if } B \ll 1 \end{cases} \qquad B = \frac{3\pi\Sigma_0 R_0^2}{M_{\rm p}}$$

DISC MIGRATION: SUMMARY

- Type I migration timescales are very short ($\sim 10^4$ yrs).
- Type II migration is 1-2 orders of magnitude longer.
- Suggests that planets should all fall into the star within the lifetime of the disc.



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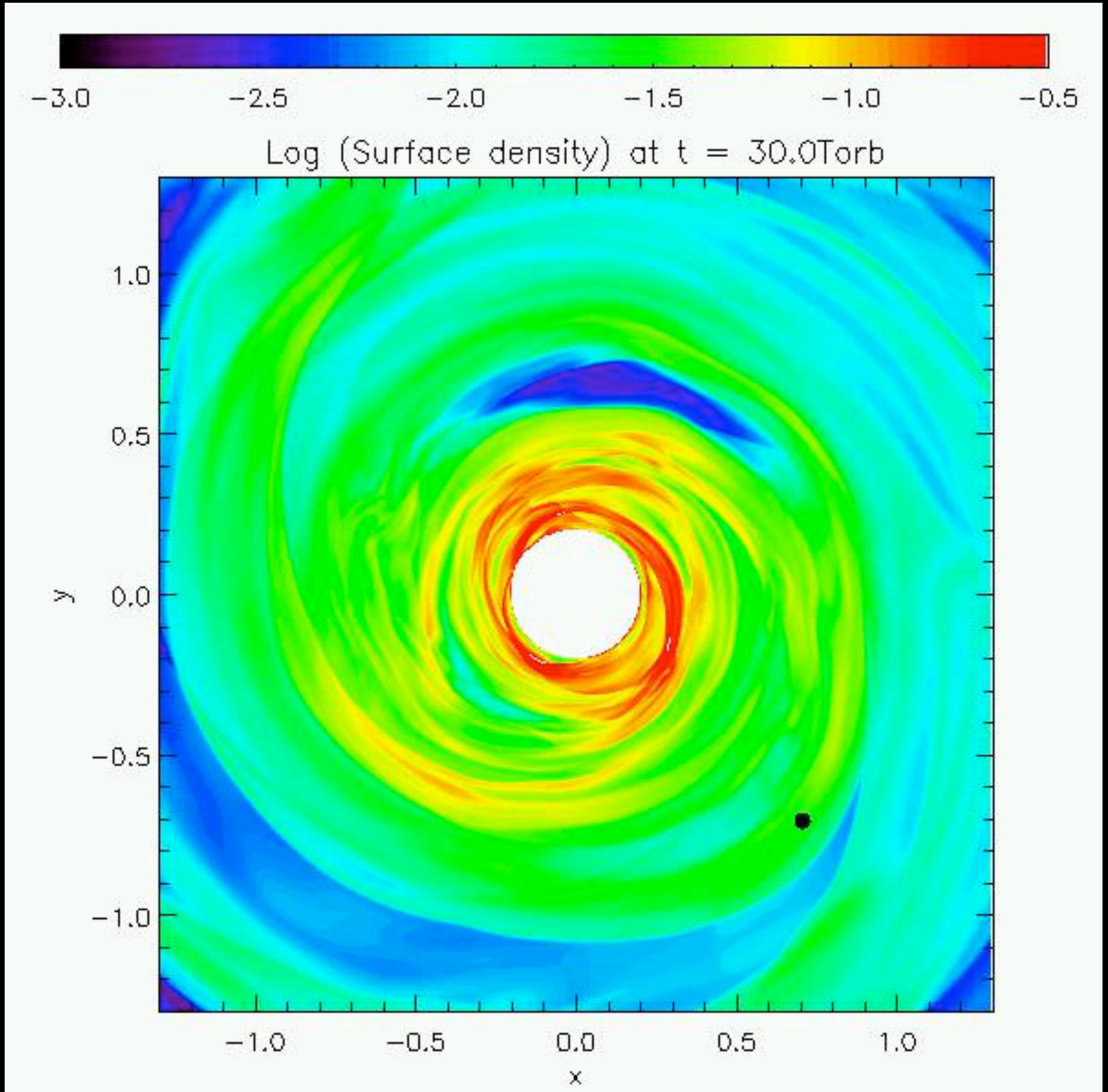

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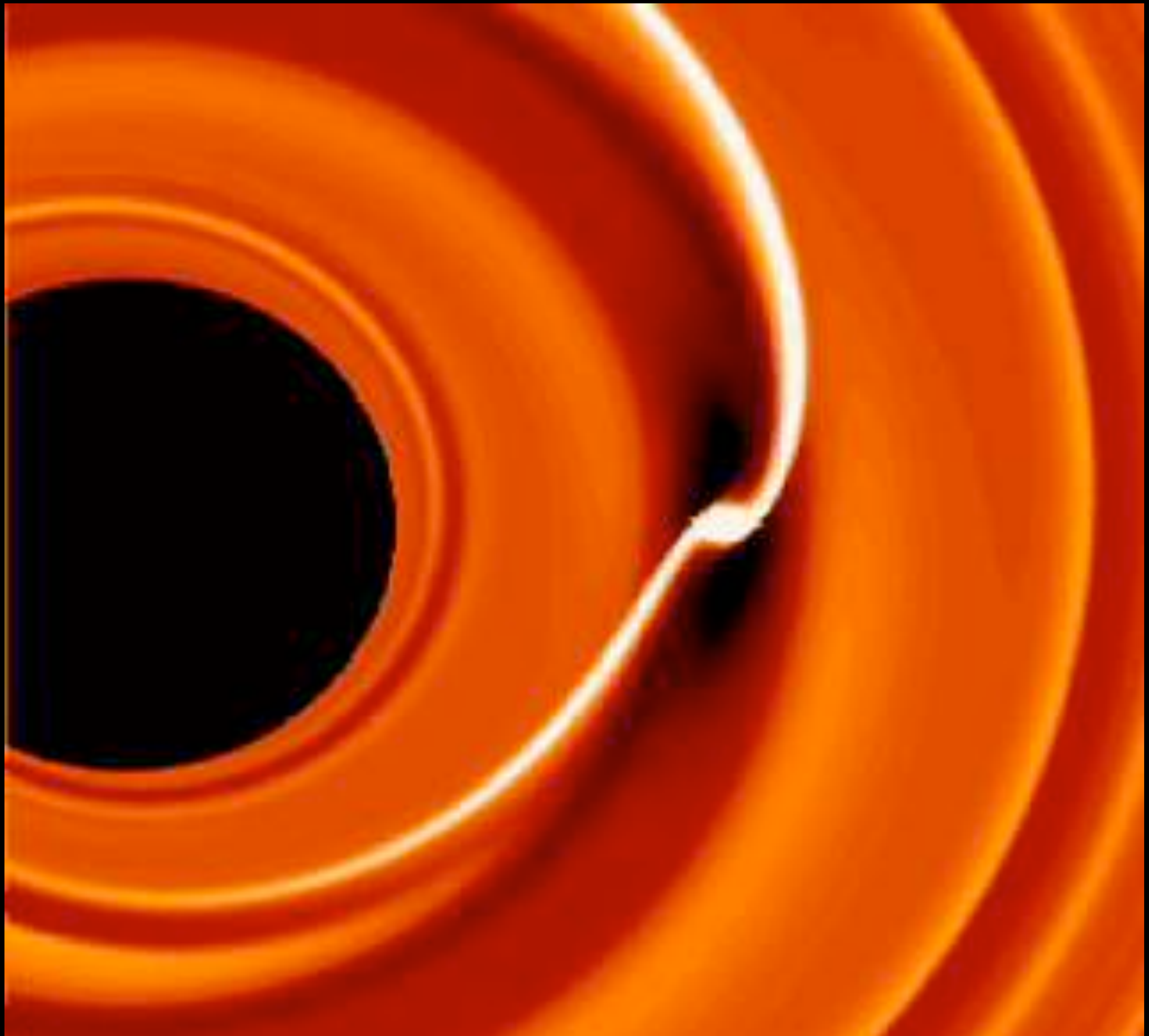

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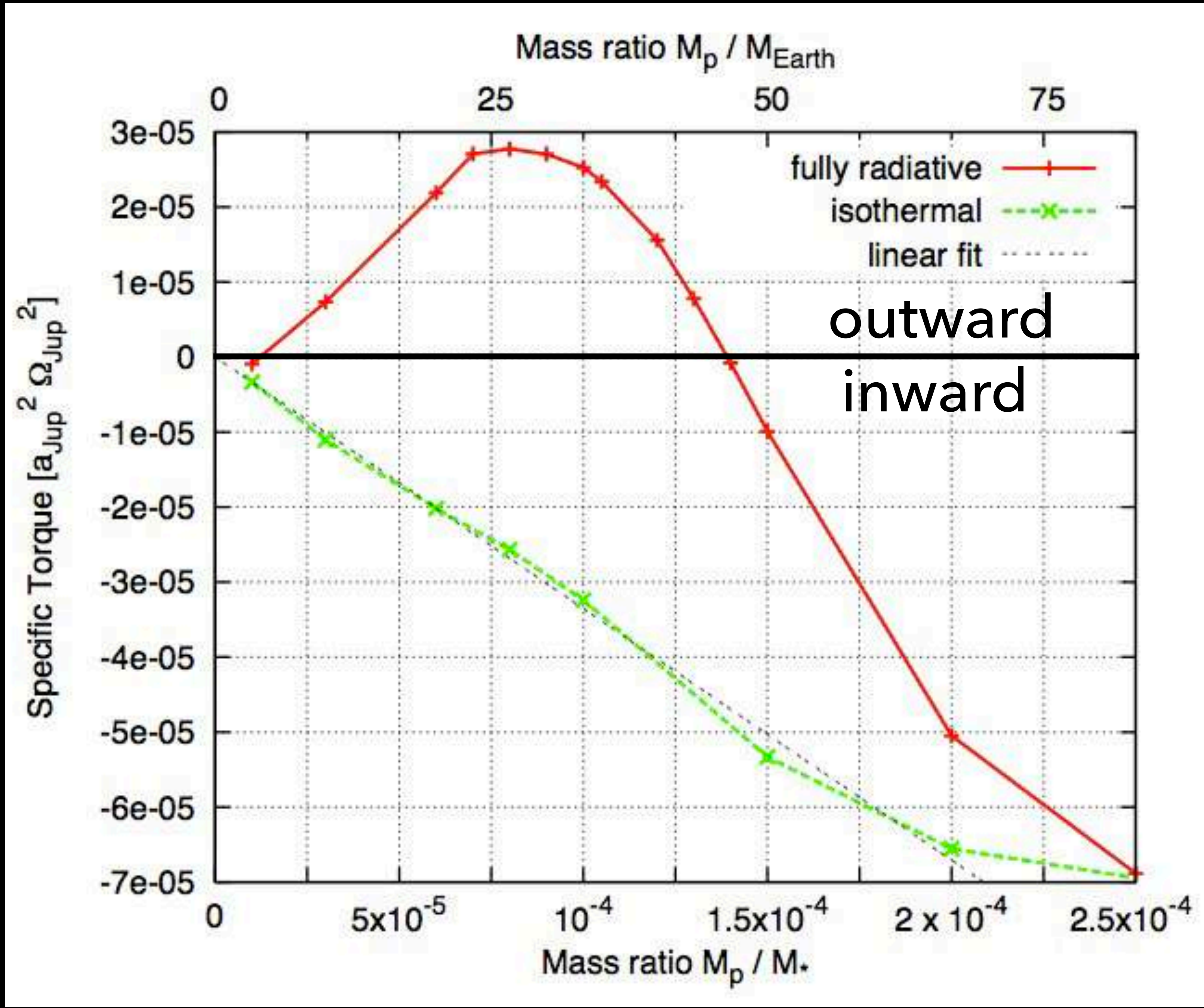
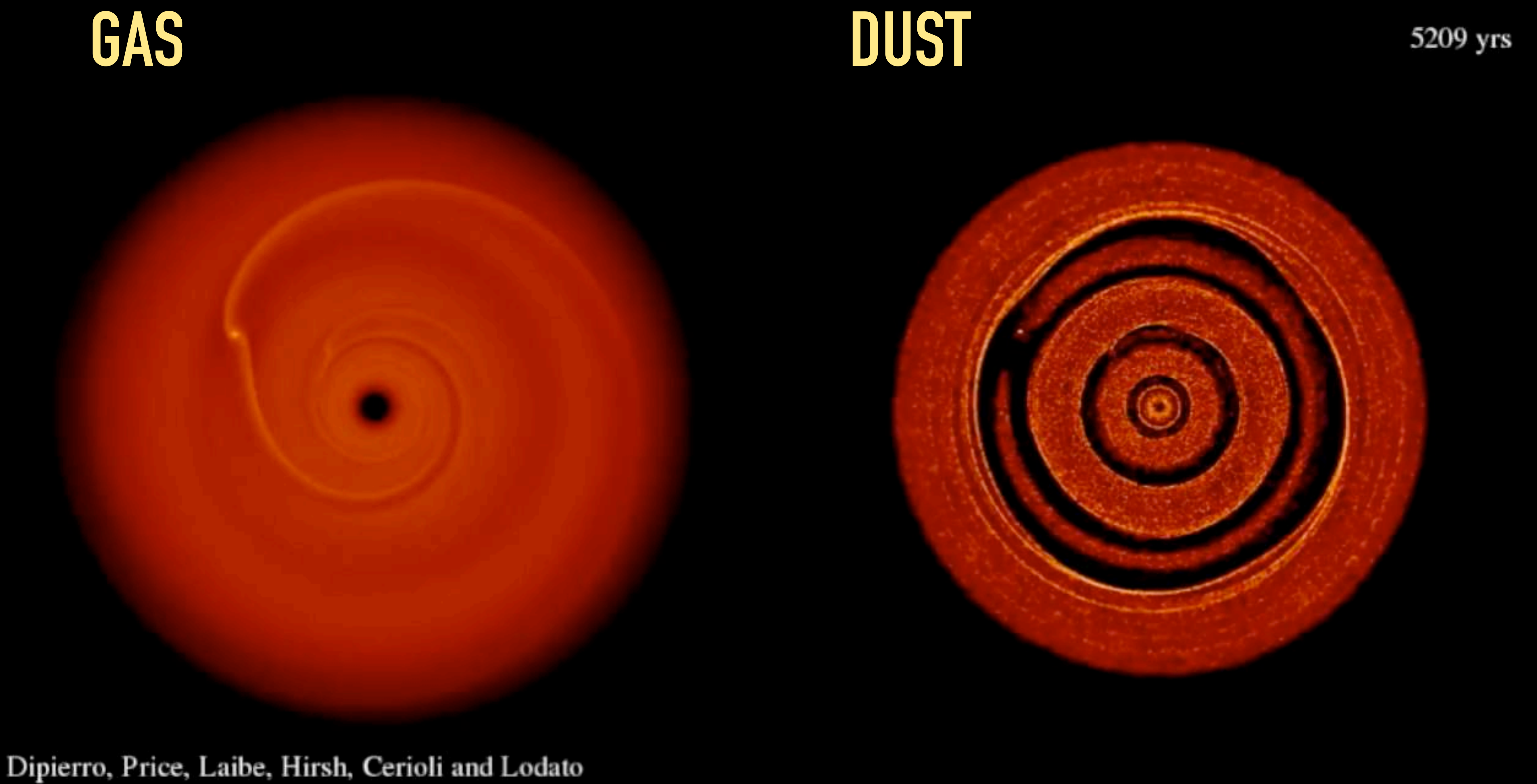
- Type II migration is 1-2 orders of magnitude longer.
- Suggests that planets should all fall into the star within the lifetime of the disc.
- Not as effective in gravito-turbulent discs or for asymmetric gaps (Type III migration).
- Thermodynamics is an important factor

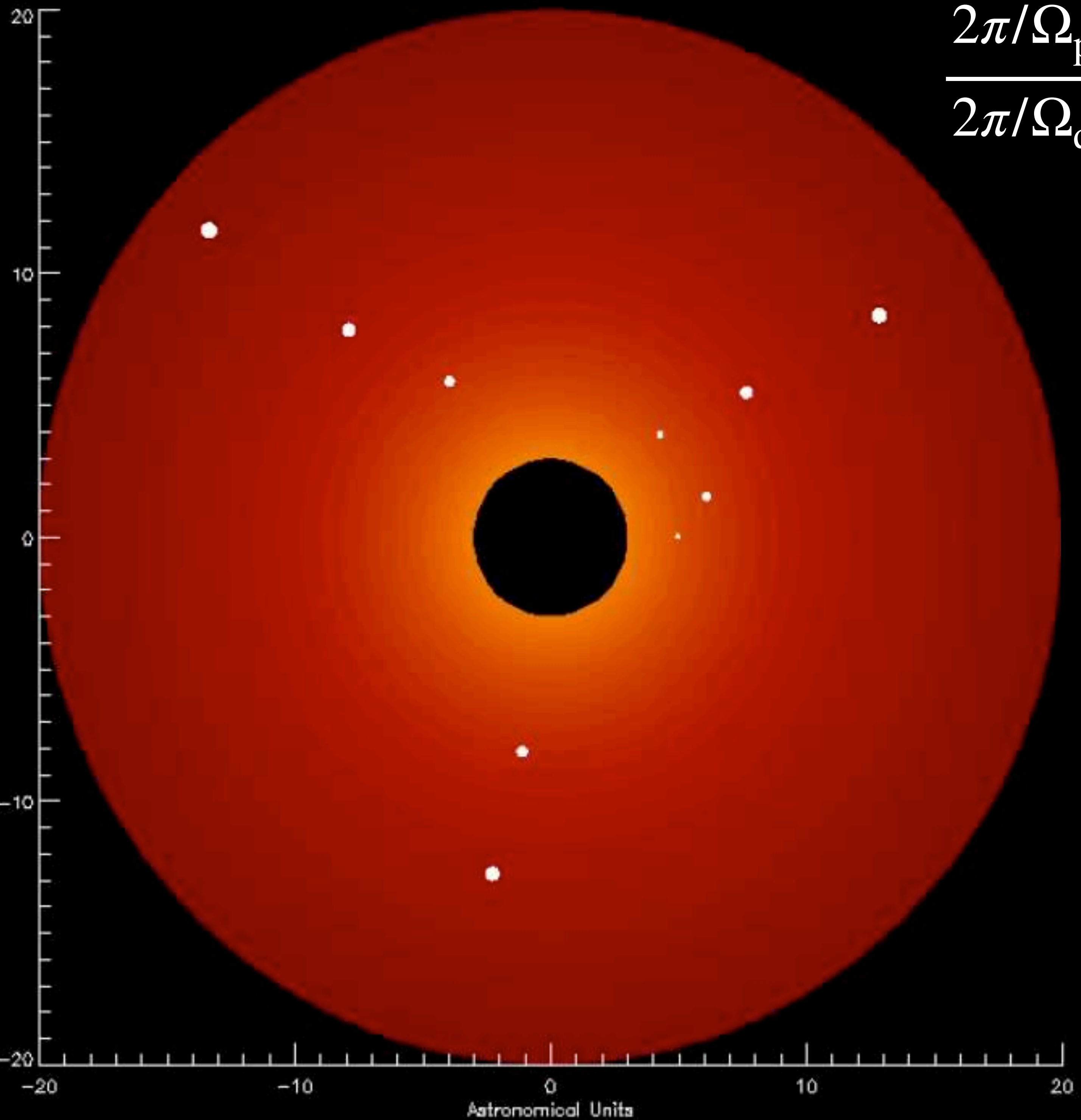


EFFECTS ON DUST



- Plant gaps prevent large dust grains from migrating past the planet. Only small grains that are well coupled to the gas can still cross. Can affect the grain size distribution in the inner disc.

MEAN MOTION RESONANCES (MMR)

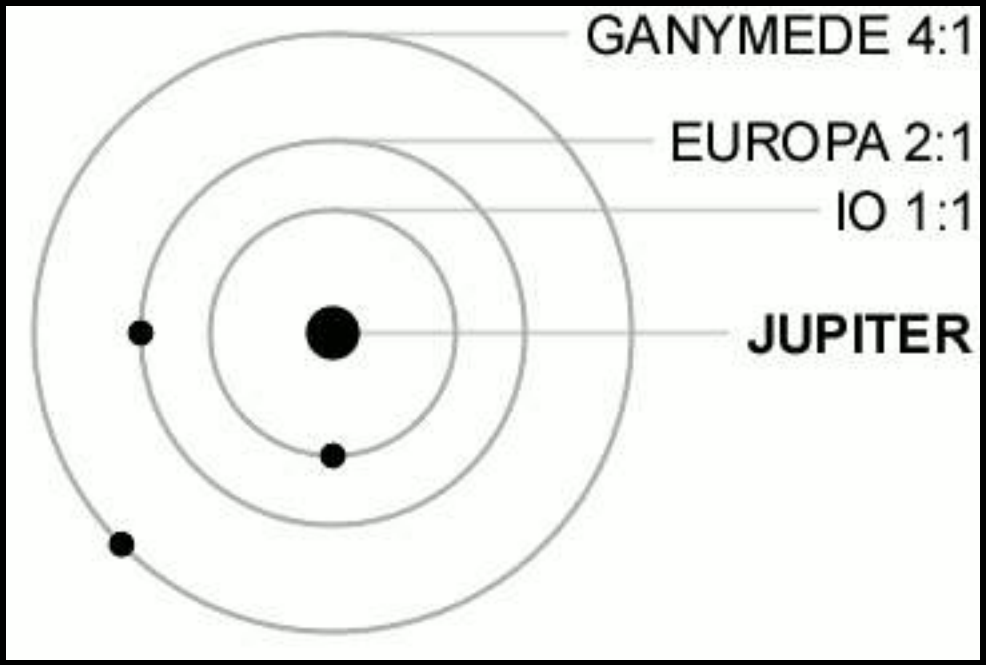


$$\frac{2\pi/\Omega_p}{2\pi/\Omega_q} = \frac{P_p}{P_q} = \frac{n}{m} \quad \text{where } [n, m] = 1, 2, 3, \ldots$$

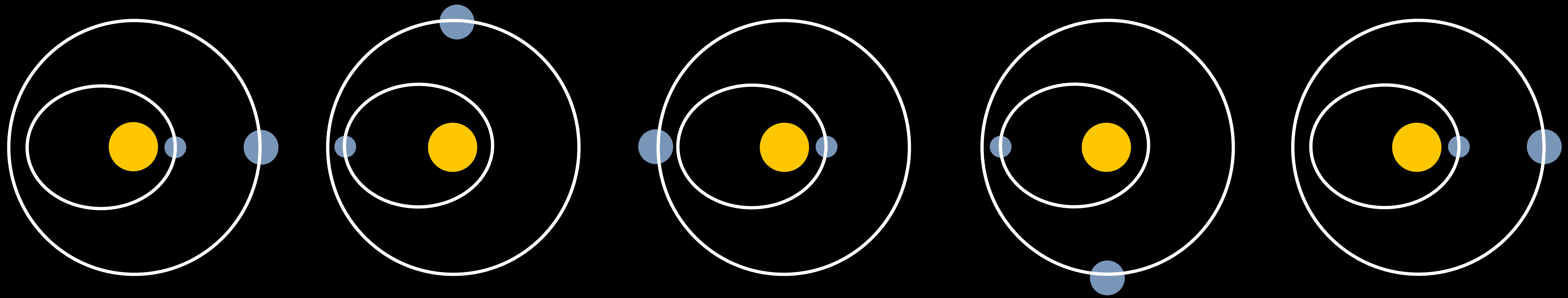
- Multiple planet systems often form resonant chains as inner planets catch migrating planets behind them.

MEAN MOTION RESONANCES (MMR)

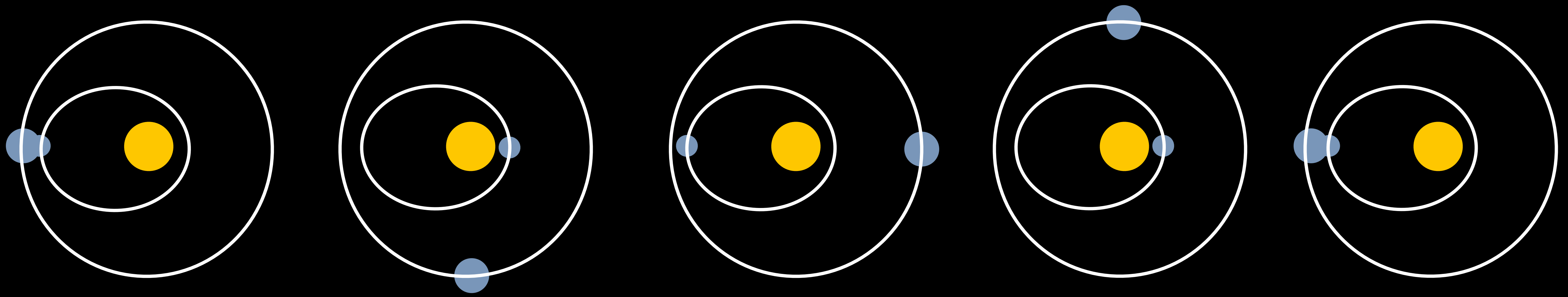
- Similar to finding the natural frequency on a swing. Pushing at the right time is important for stable motion.



Stable

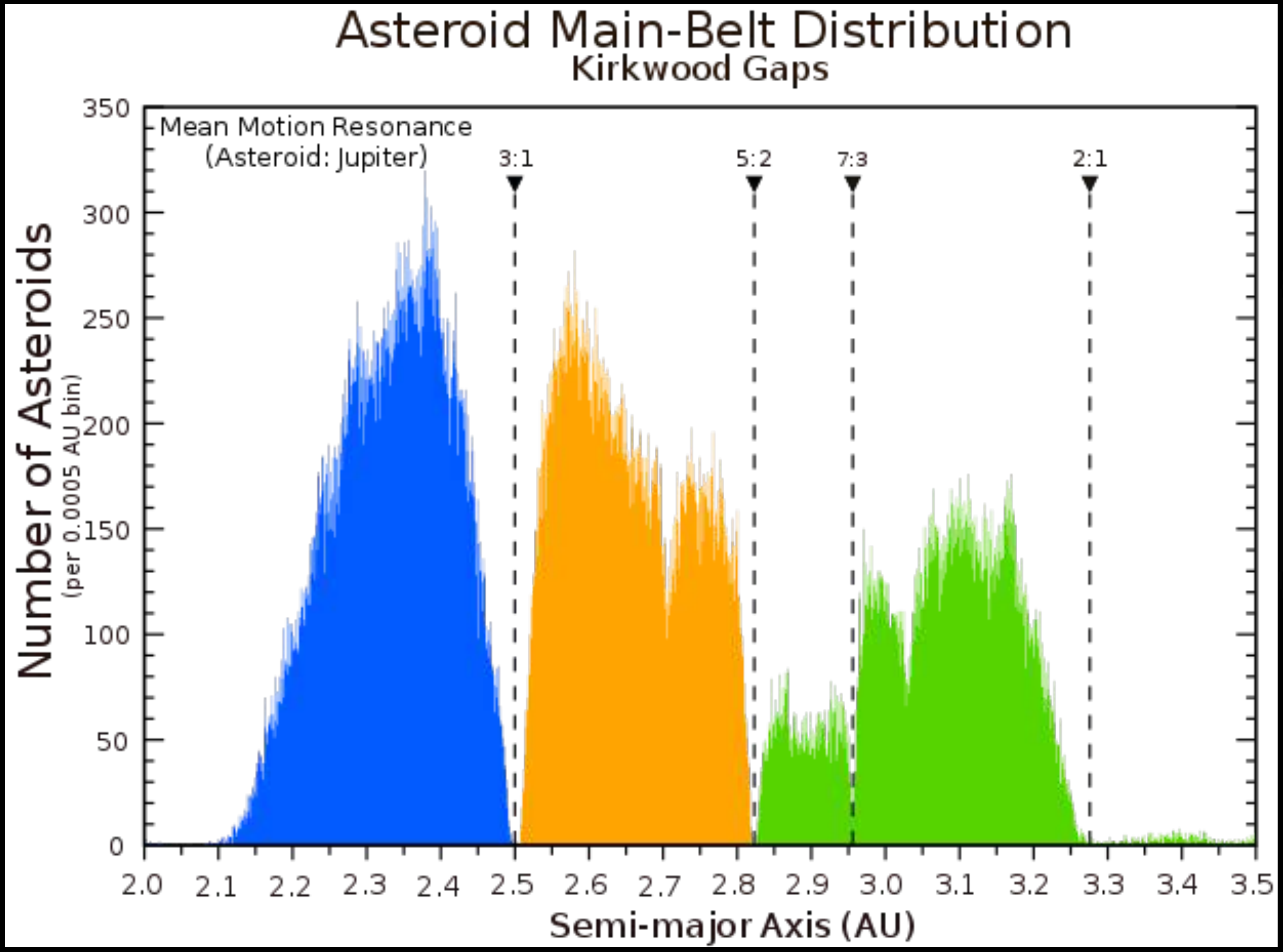


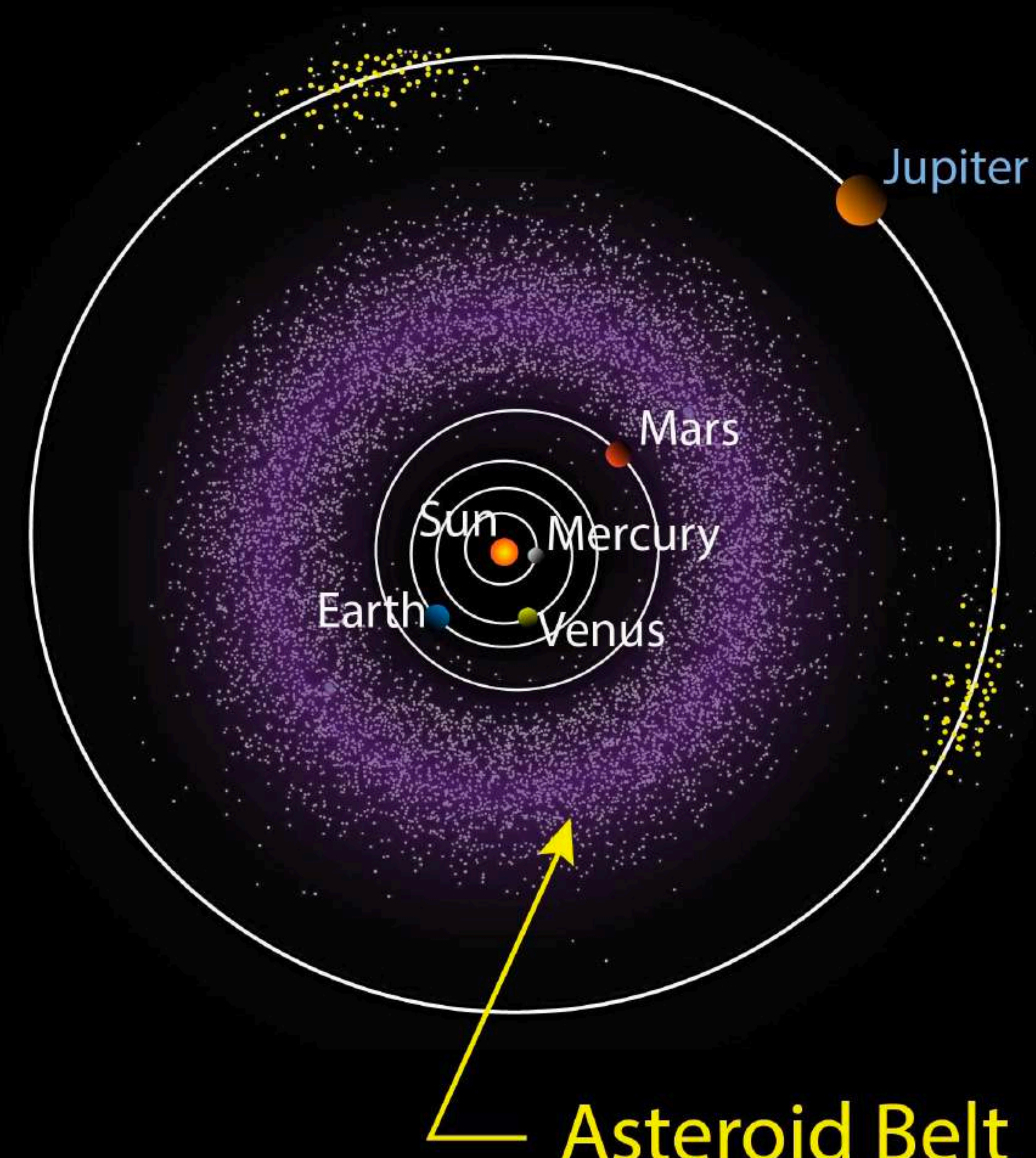
Unstable

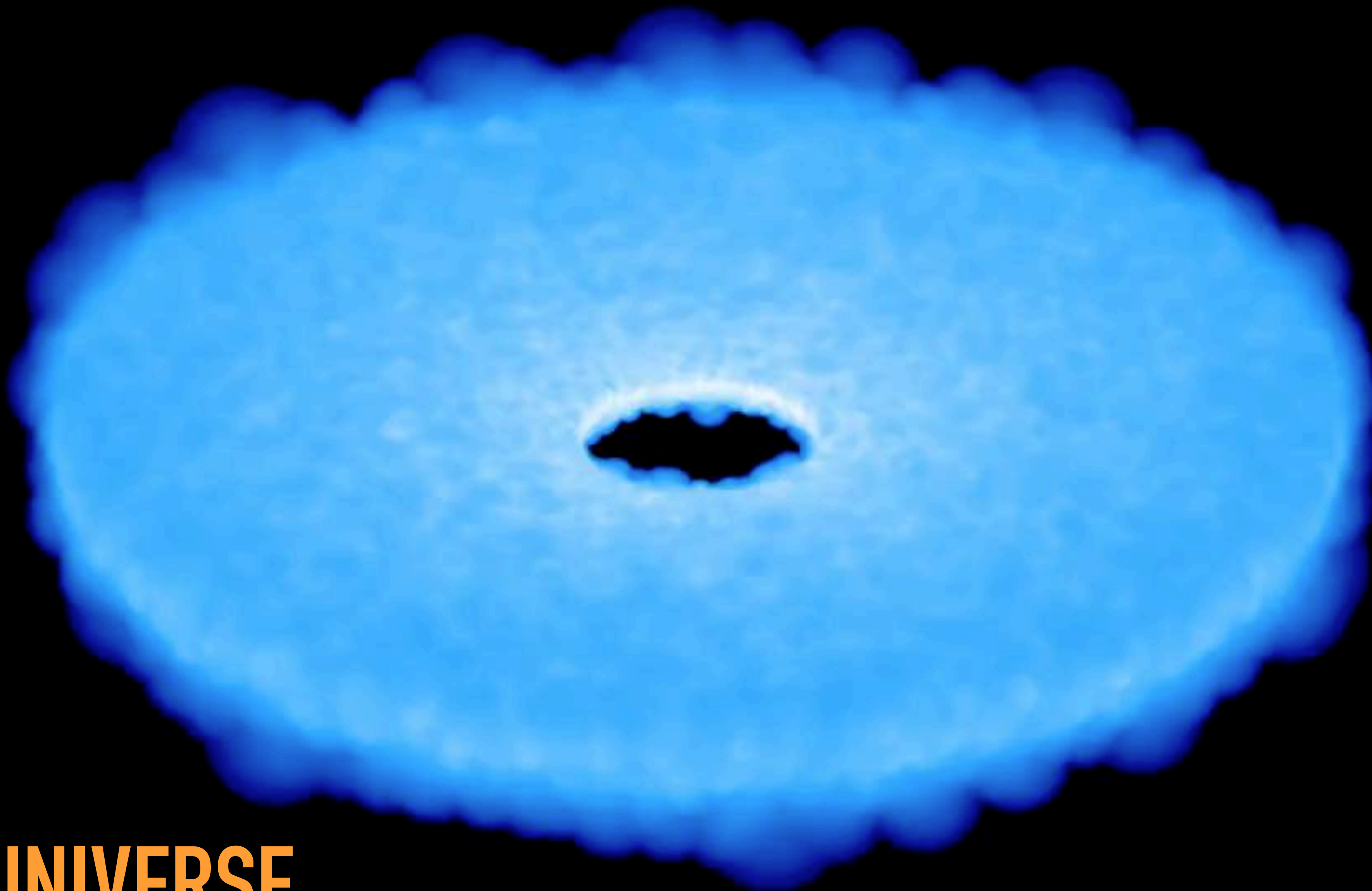


MEAN MOTION RESONANCES (MMR)

- Structure in the astroid belt revealed when plotting the mean orbits (instantaneous snapshots don't show this because of the random eccentricities of the astroids). Similar effect in Saturn's rings.







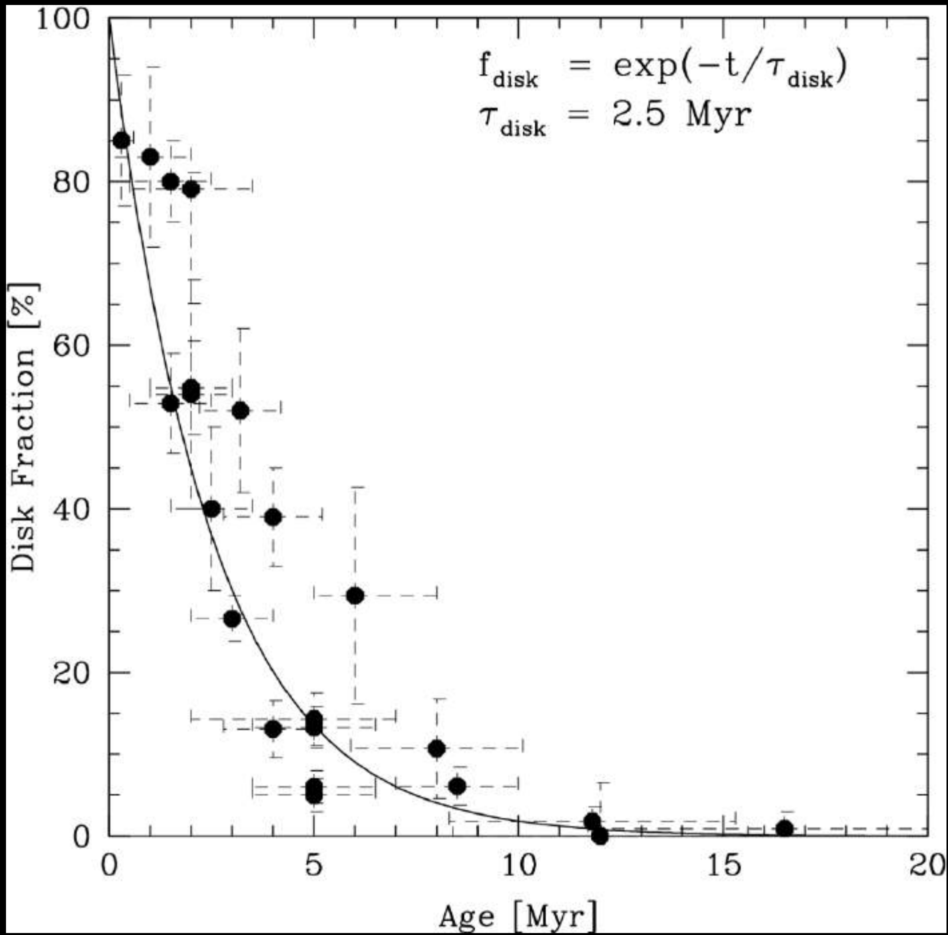
FROM UNIVERSE

TO PLANETS

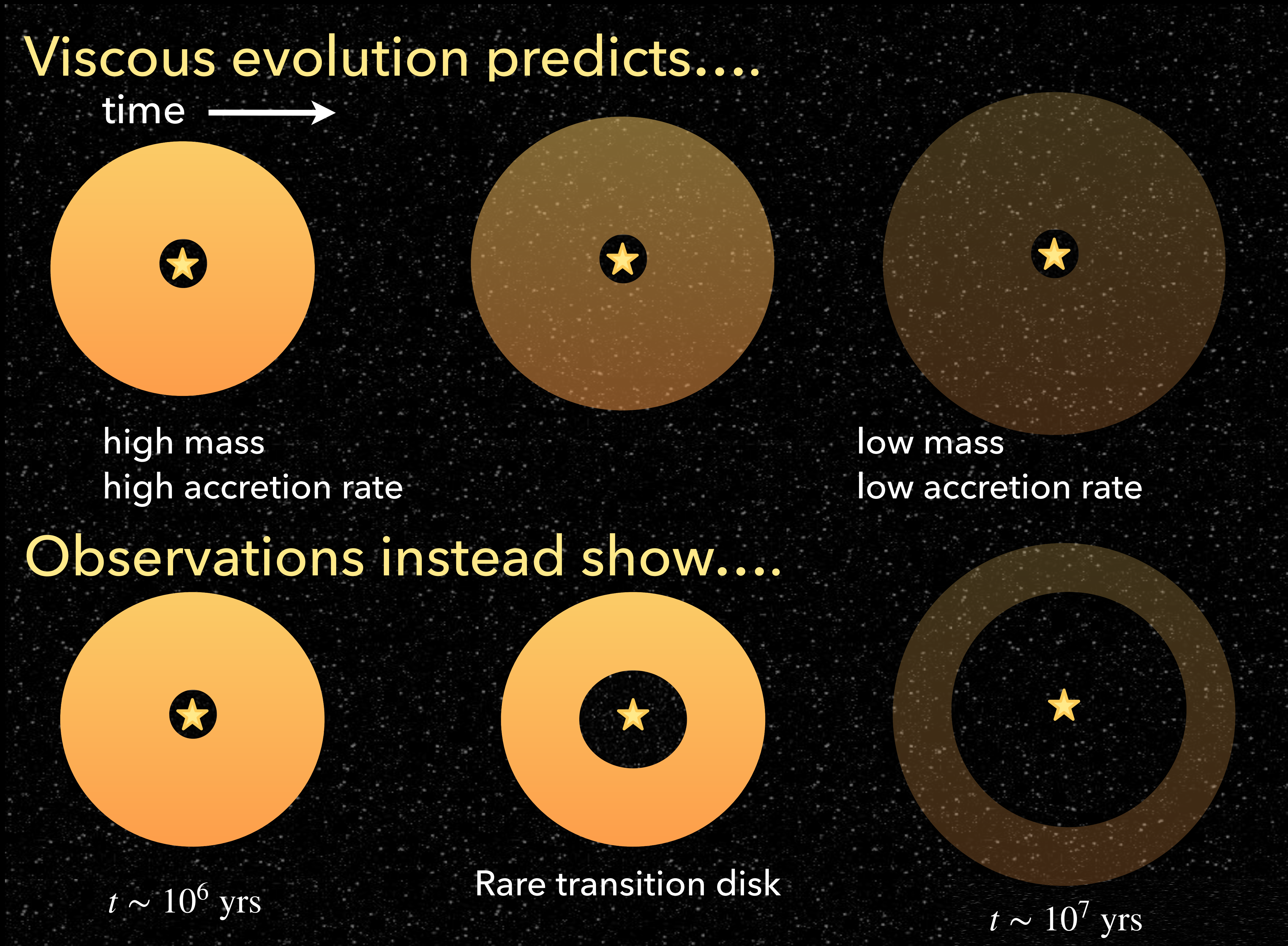
LECTURE 4.3: PHOTOEVAPORATION

VISCOUS EVOLUTION

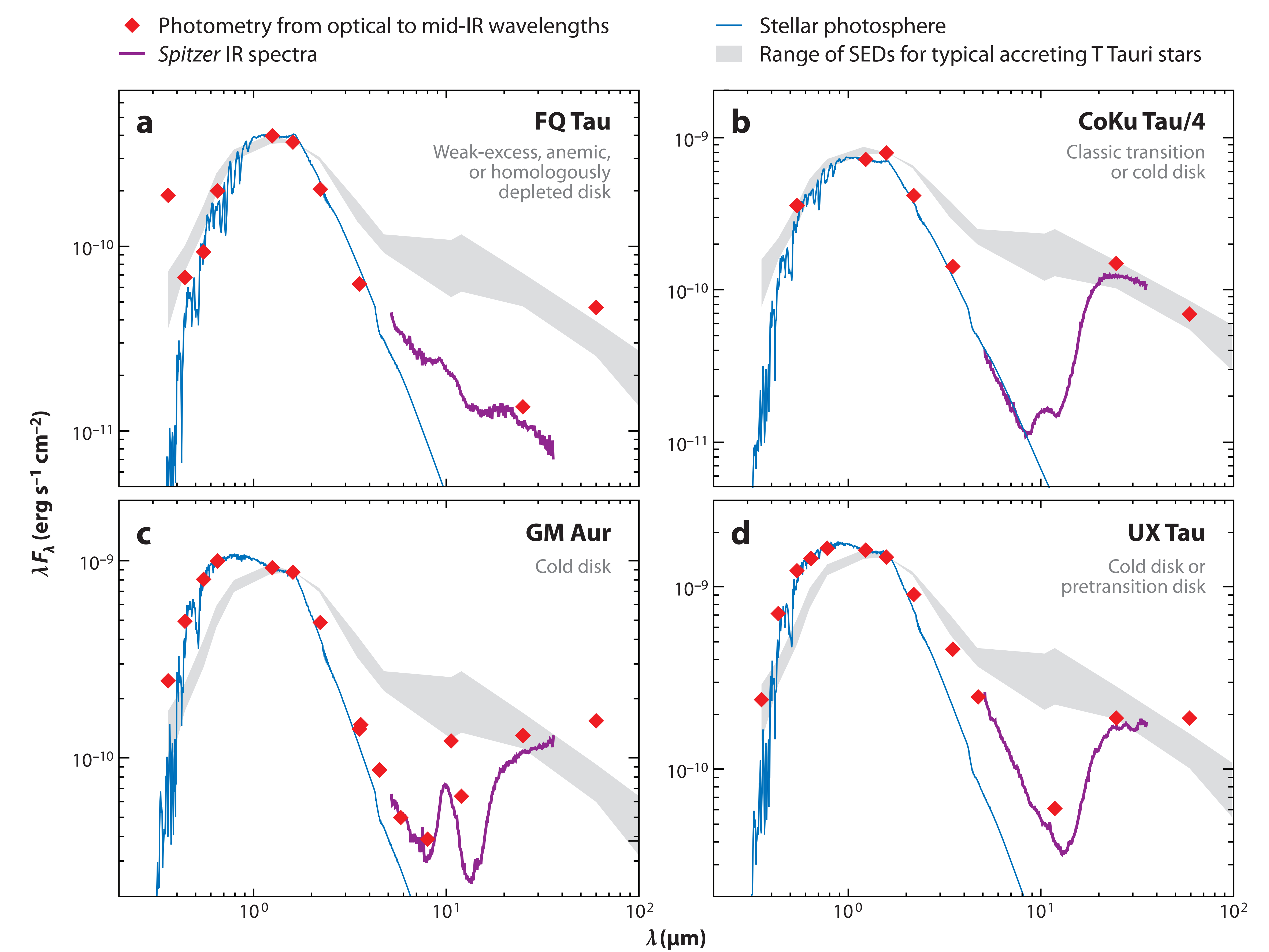
- Viscous evolution theory predicts:
  - Long disc lifetimes (~10–100 of Myr).
  - Discs should go progressively optically thin at all radii due to viscous accretion and spreading.
  - We should see many discs in “transition” phase.
- Observations show:
  - Discs are dispersed in ~1–10 Myr, with a median of 3–5 Myr.
  - Very few transition discs (~10%).
  - Clearing must be fast (~0.5 Myrs).



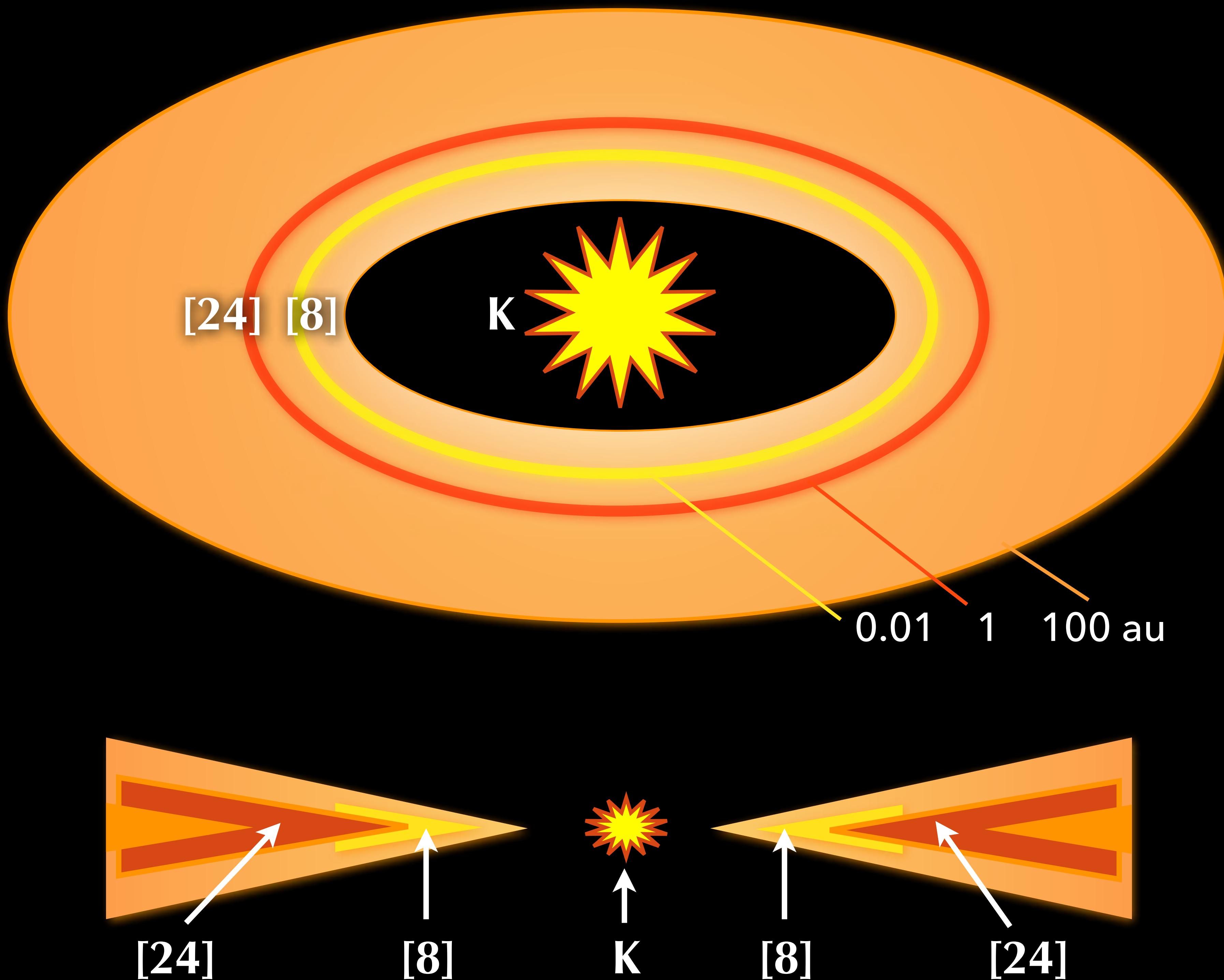
VISCOUS EVOLUTION



TRANSITION DISCS



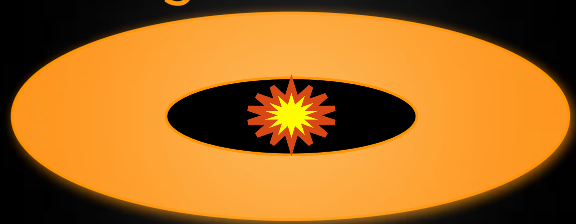
TRANSITION DISCS



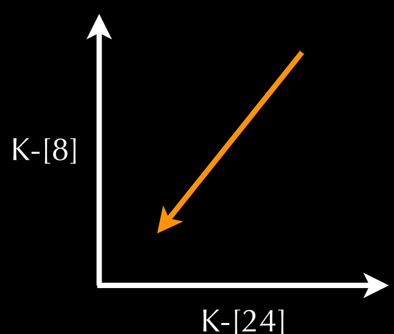
$$E \sim T \sim \nu \sim 1/\lambda \longrightarrow \begin{array}{l} T_{[8]} \approx 1800 \text{ K} \\ T_{[24]} \approx 600 \text{ K} \end{array}$$

TRANSITION DISCS

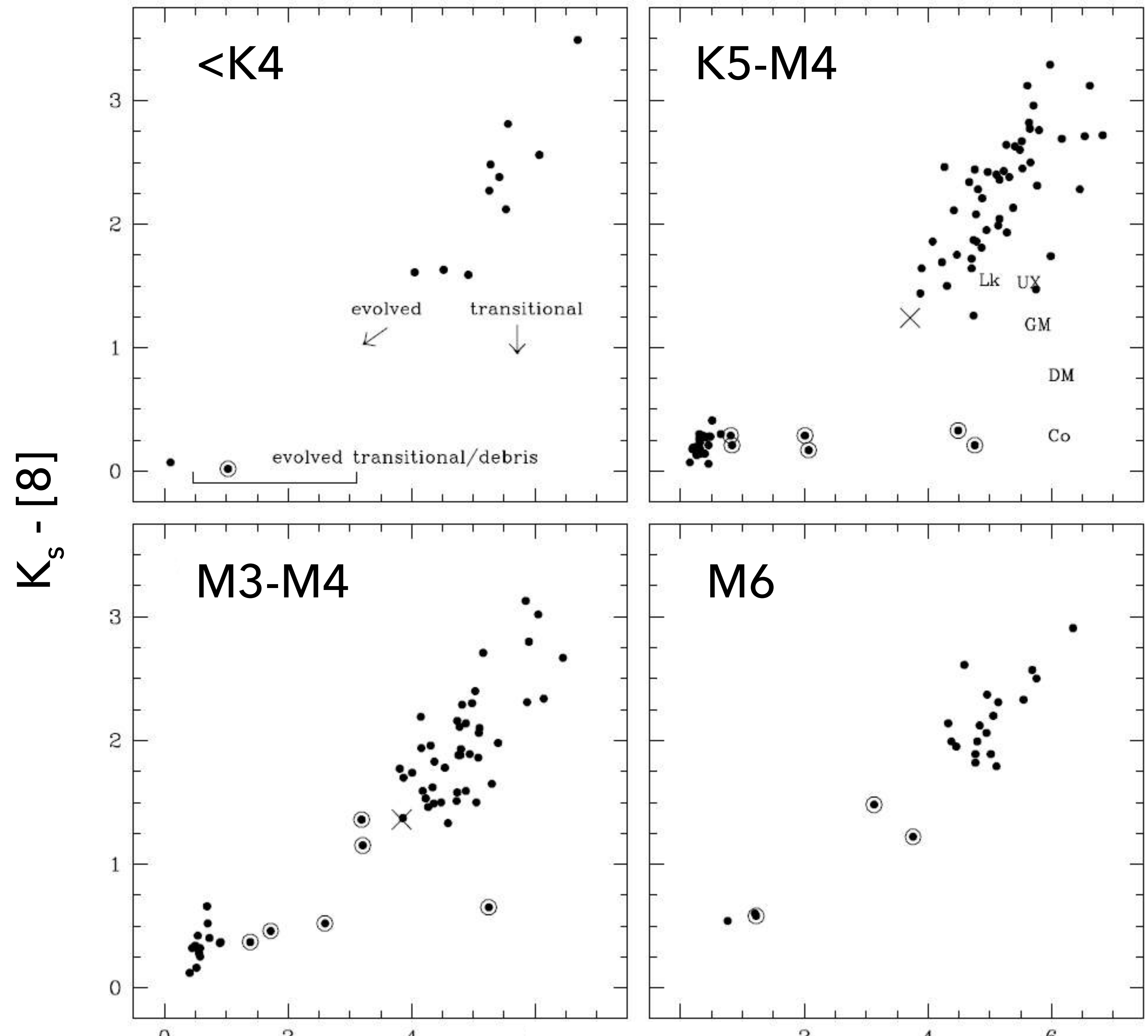
Homogeneous draining



- Equal decrease in K-[8] and K-[24]



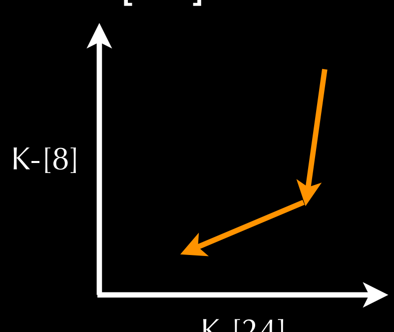
Two-colour diagram of young sources in Taurus



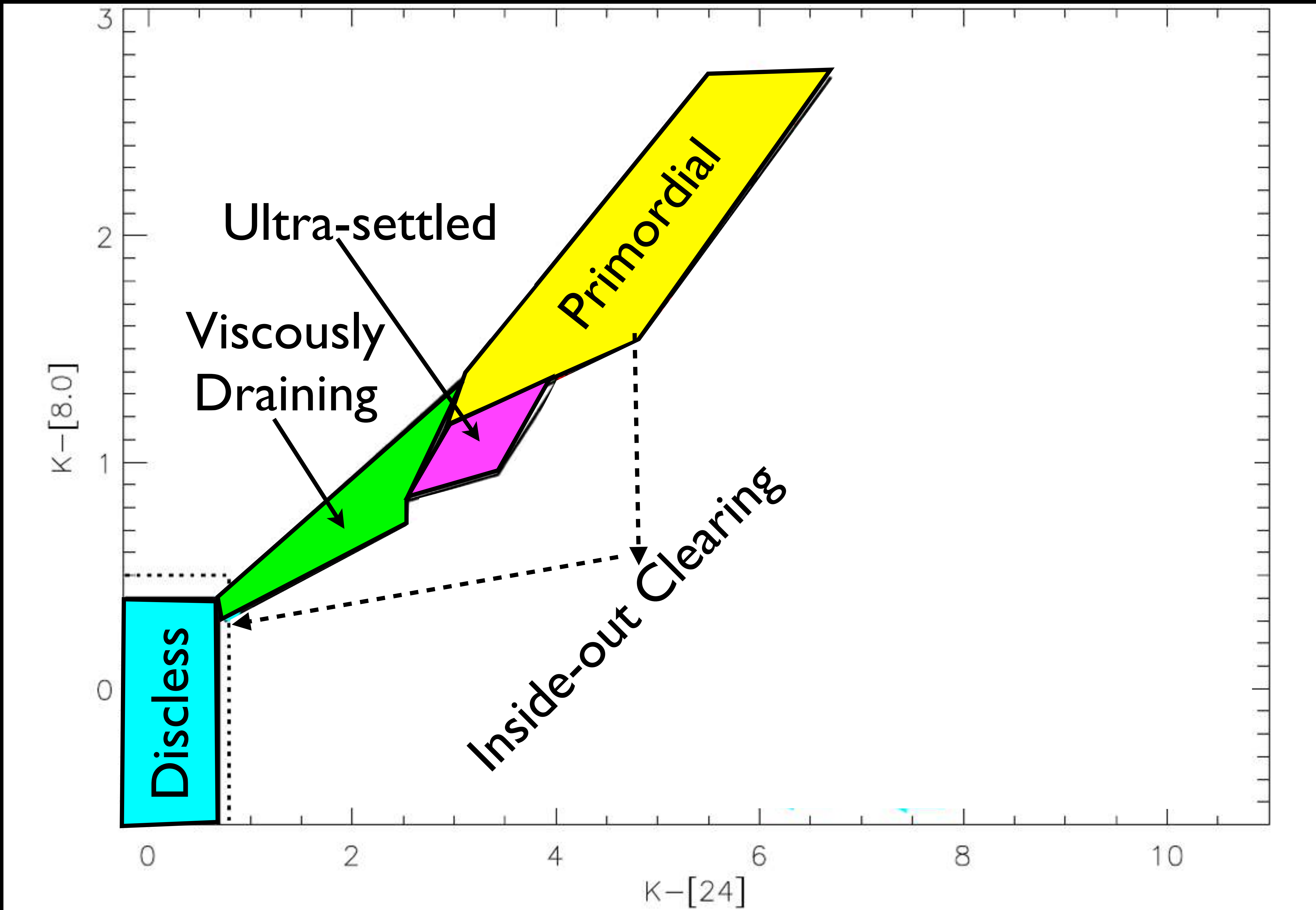
Luhman et al. (2010)

Inside-out draining

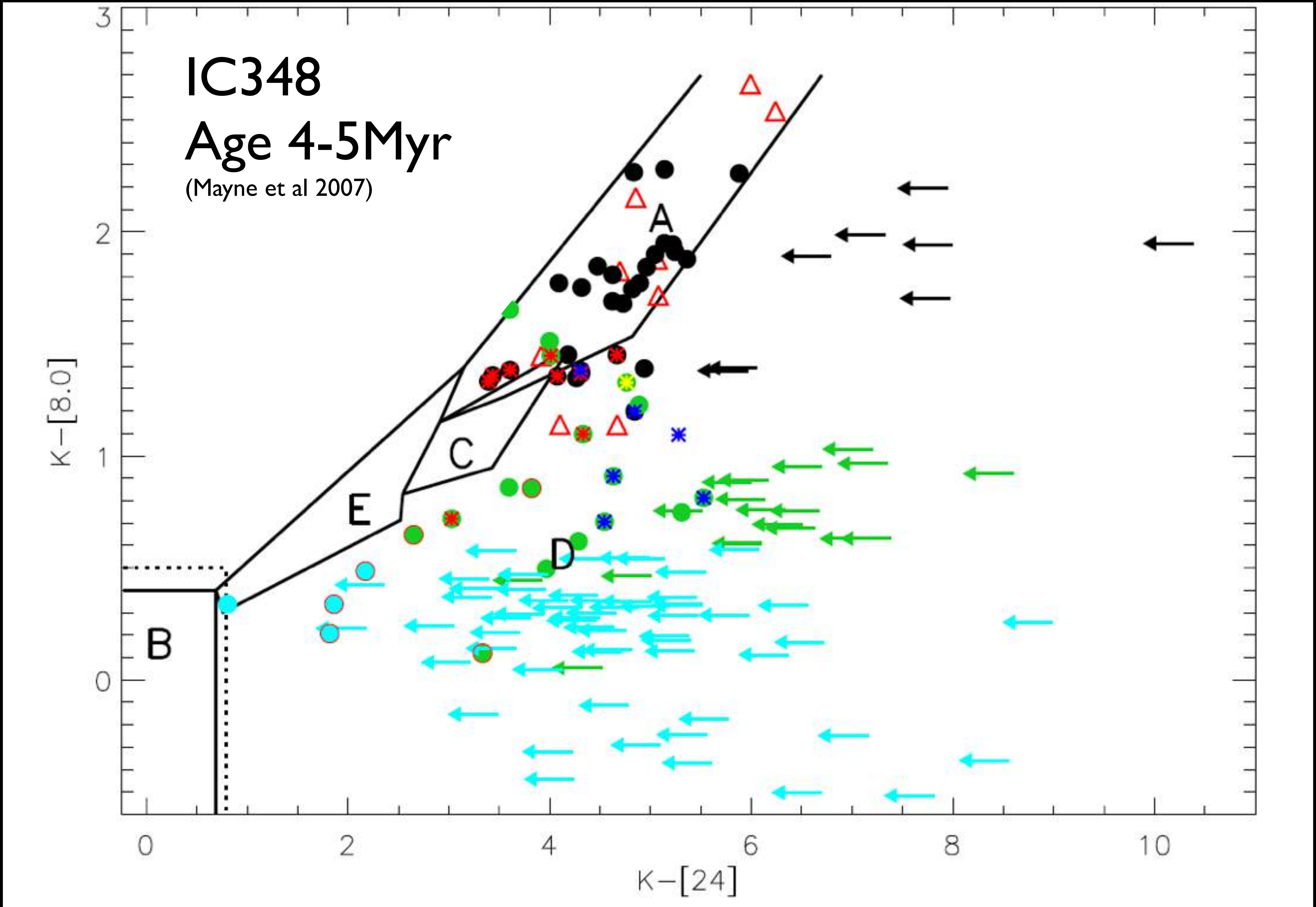
- K-[8] decreases fast
- K-[24] decreases slow



TRANSITION DISCS



TRANSITION DISCS



PHOTOEVAPORATION: EUV (13.6–100 eV)

- Thermal winds are produced by absorption of high energy radiation. Gas can escape if $c_s > v_{esc}$.

- Naively:

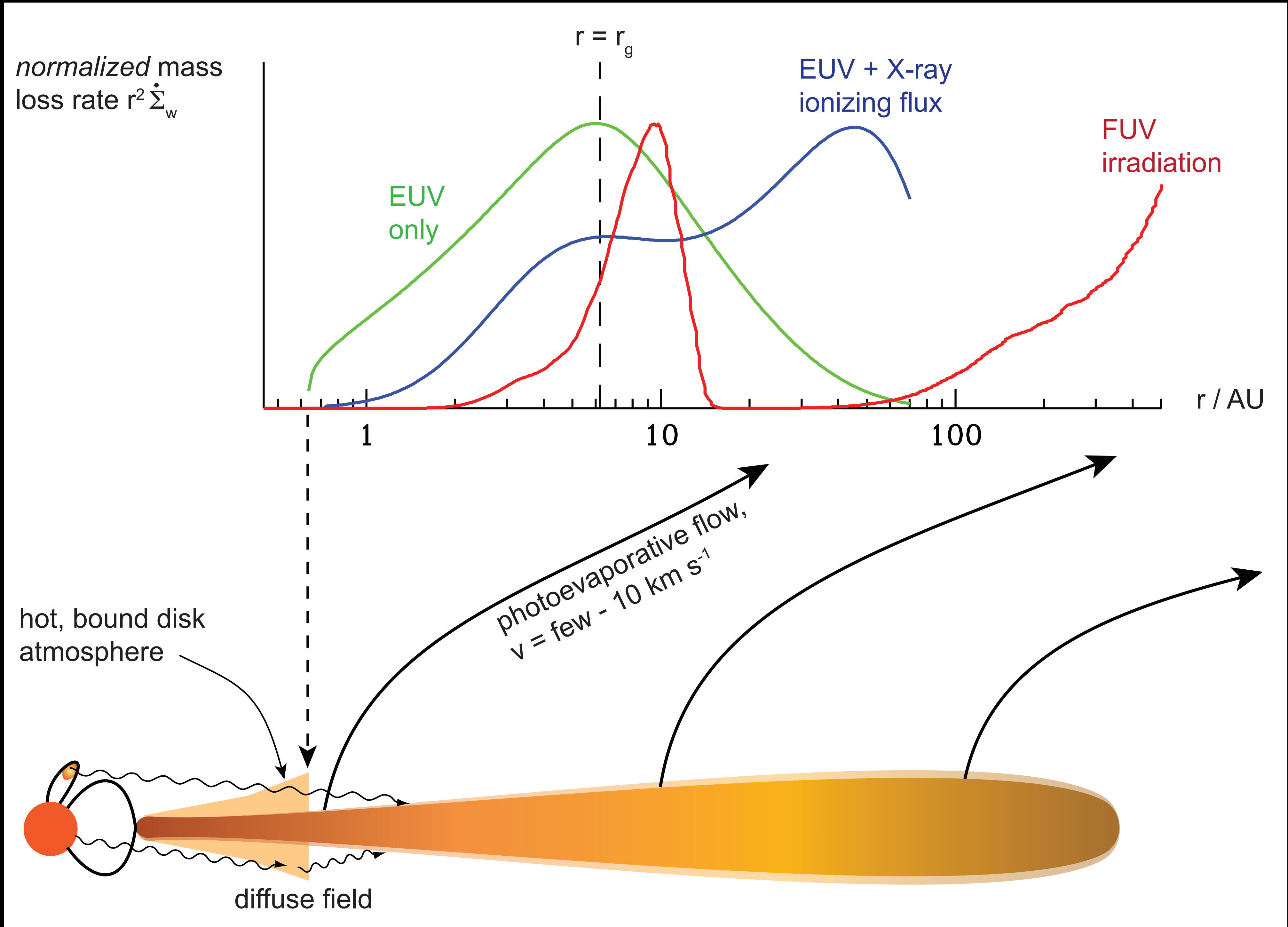
$$R_g = \frac{GM_*}{c_s^2}$$

$$\sim 5\text{–}10 \text{ au}$$

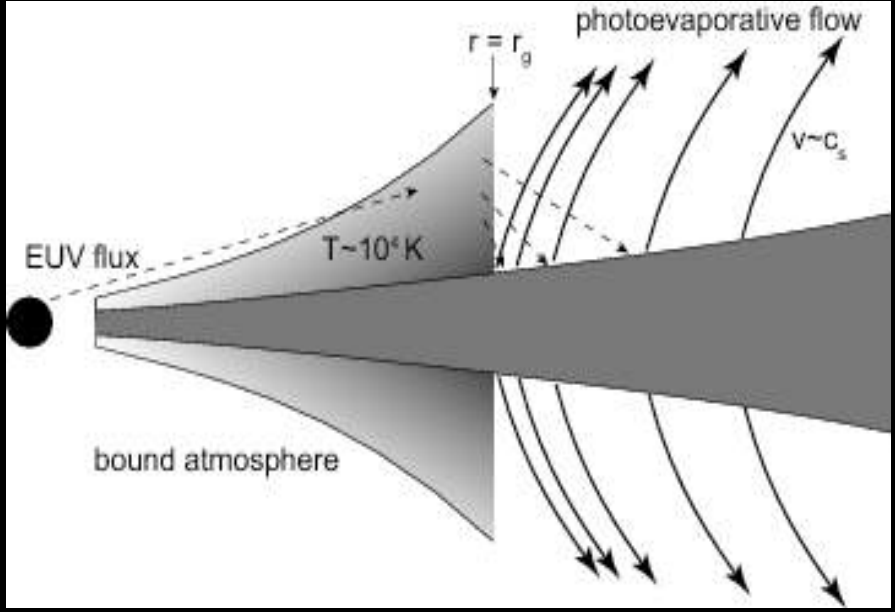
- Bernoulli flow:

$$R_c \approx \frac{R_g}{5}$$

$$\sim 1\text{–}2 \text{ au}$$



PHOTOEVAPORATION: EUV (13.6–100 eV)



- Ionises H at the disc surface ($T \sim 10^4$ K, $c_\mathrm{s} \sim 10\ \mathrm{km/s}$; negligible disc heating).
- Large absorption cross-section: peaks at 13.6 eV (drops as ν^{-3}).
    - Probably blocked at early times by accretion columns or (partially-) neutral X-ray/FUV winds.
- Dominated by diffuse recombination (~1/3 of H recombinations produce another ionising photon).
    - Mass loss is localised near R_g.
    - Dispersal times are largely dependent on viscous accretion.

$$\dot{M}_\mathrm{wind} \approx 4 \times 10^{-10} \left(\frac{\Phi}{10^{41}\ \mathrm{s}^{-1}} \right) \left(\frac{M_*}{M_\odot} \right)^{1/2} M_\odot\,\mathrm{yr}^{-1}$$

PHOTOEVAPORATION: FUV (6–13.6 eV)

- Drive significant heating in the disc surface (~100-1000s K) and can penetrate ~2 orders of magnitude deeper than EUV. Mostly attenuated by dust and PAHs.
- Mass-loss rate peaks at ~5–10 au, but only dominant beyond 100 au. Important for truncating the disc.
- Many heating mechanisms → complicated chemical networks.
    - Photoelectric emission from dust grains.
    - Neutral C ionisation.
    - H_2 photodissociation.
    - FUV pumping and collisional de-excitation of vibrationally excited H_2.
    - Reformation of H_2 from FUV dissociated H atoms.
    - Gas-grain collisions (dominated by surface area of small grains).
- Gas cooling is also important and occurs through fine-structure lines in atoms and low-lying rotational lines from molecules.

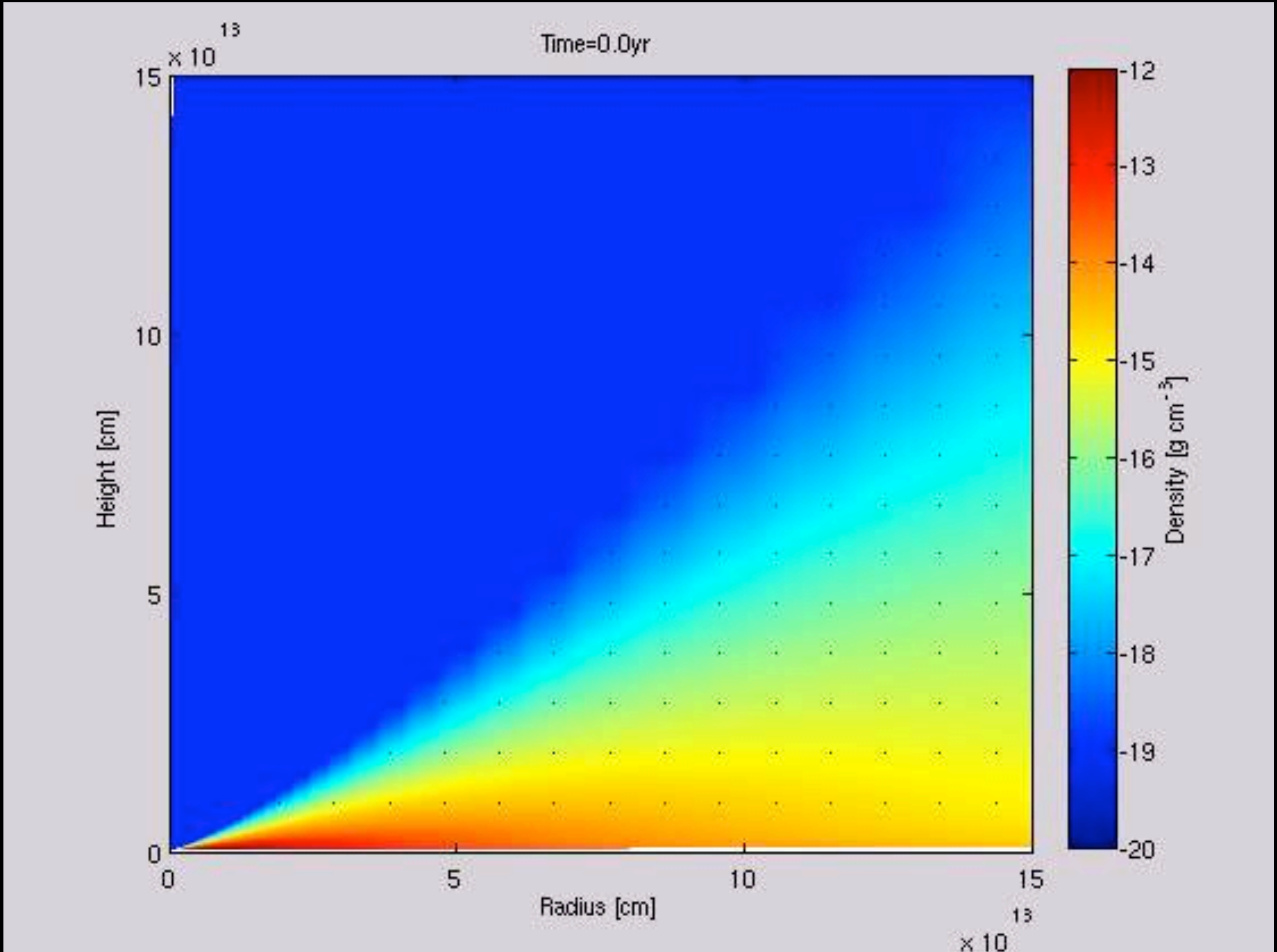
PHOTOEVAPORATION: X-RAY (0.1–1 keV)

- Young solar type stars emit 10^{-3} of their bolometric luminosity in X-ray (main sequence stars only 10^{-7}).
- Similar penetration to FUV. Base flows have $T \sim 3000$–5000 K and are $\sim 40 \times$ larger than EUV rates ($\gtrsim 0.3 - 0.4$ keV; harder X-rays penetrate too deep to drive winds, but important for heating/ ionising mid-plane).
- Spectral range and high energies → ionisation of inner (K-shell) electrons in heavier elements (primarily in O, but also C and Fe).
  - Gas primary heated by subsequent release of photo/Auger electrons that collisionally ionise and/or heat the lighter atoms/molecules in the disc.
- Cooling mainly through metal line emission and collisions with colder dust grains.

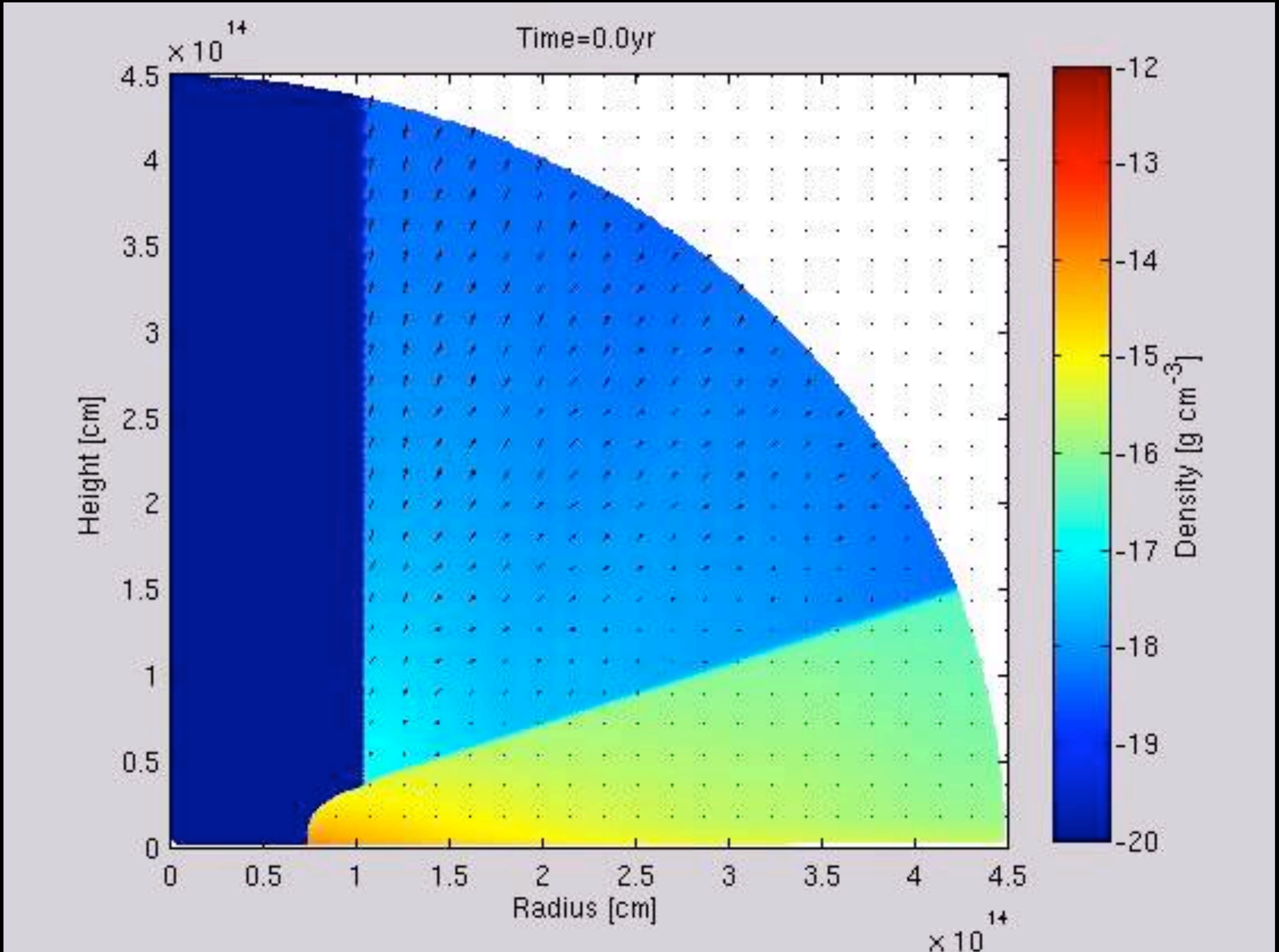
PHOTOEVAPORATION

- X-EUV radiation ionises and heats the disc atmosphere:
  - Bound atmosphere at $R < R_{\mathrm{g}}$.
  - Thermal wind at $R > R_{\mathrm{g}}$ (large portion of the disc). Total mass loss rates $\sim 10^{-10}$–$10^{-8}\ M_{\odot}\ \mathrm{yr}^{-1}$ (can be comparable to viscous accretion).
- Once accretion rates drop below the wind mass-loss rate at a given radius, a gap opens (typically near R_{g}).
  - The outer and the inner disc become decoupled. The inner disc is starved.
- Inner disc (viscously) drains rapidly onto the star producing a transition disc. Direct EUV and X-ray flux photoevaporate the outer disc from the inside out.

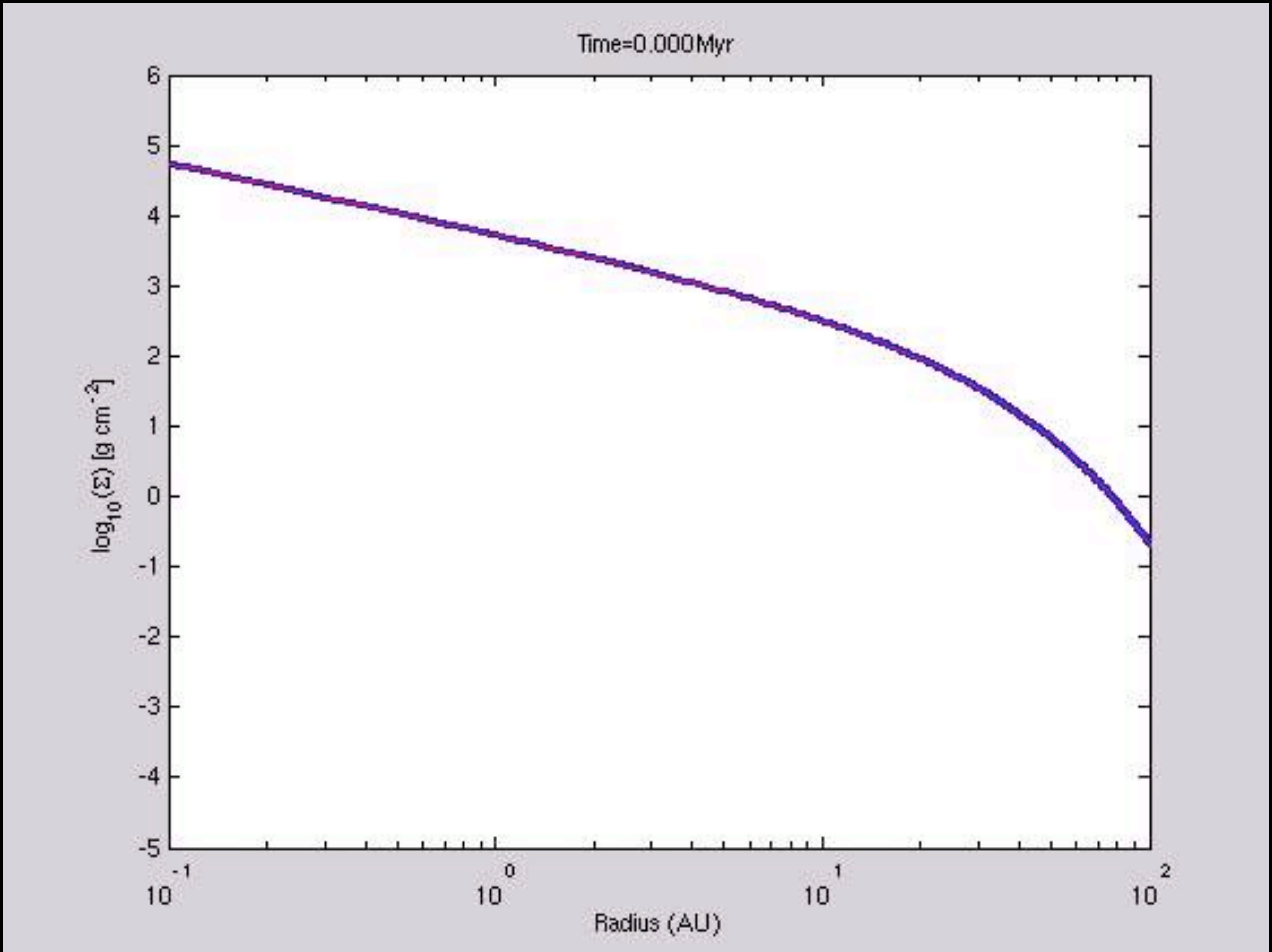
PHOTOEVAPORATION



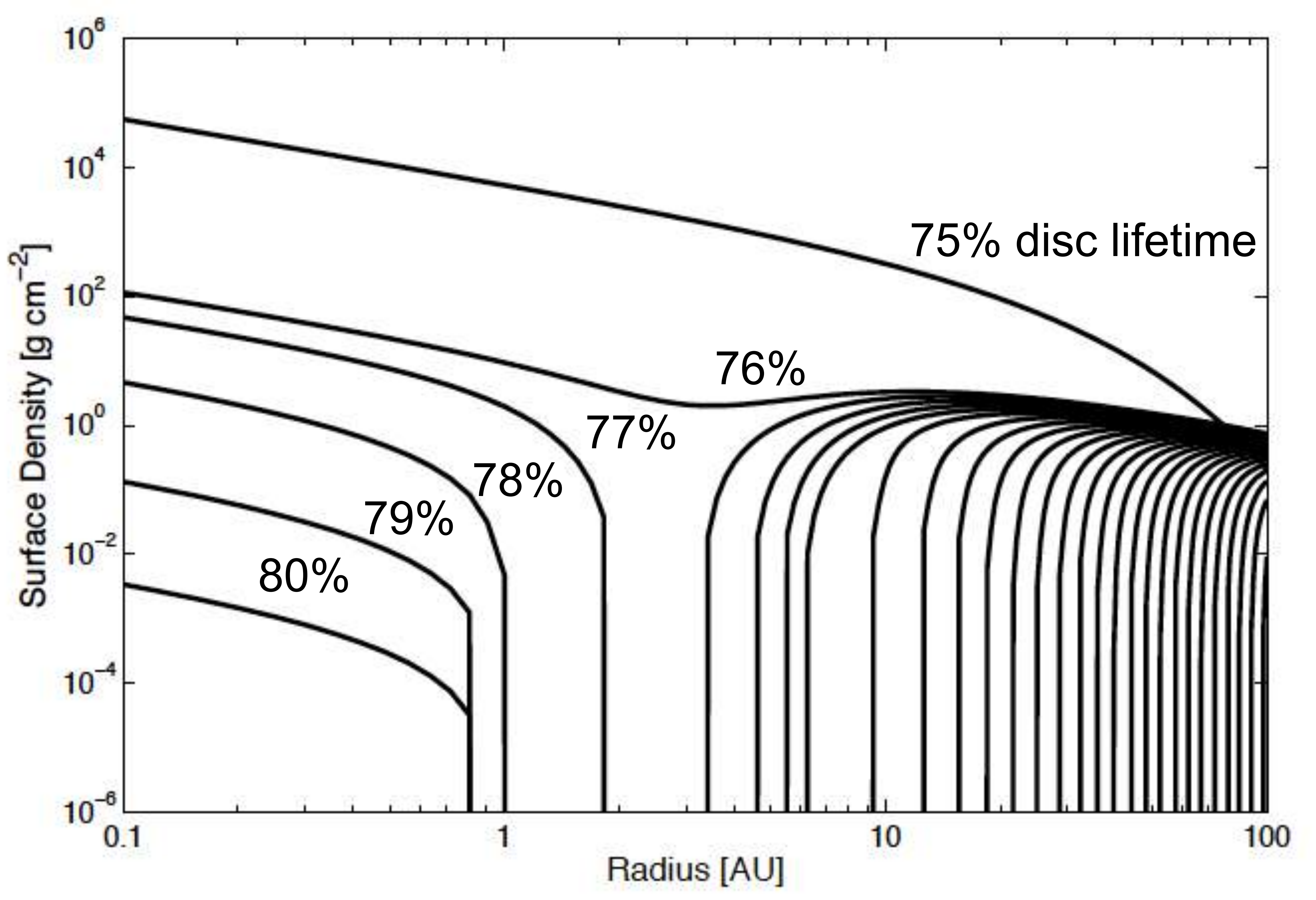
PHOTOEVAPORATION



PHOTOEVAPORATION

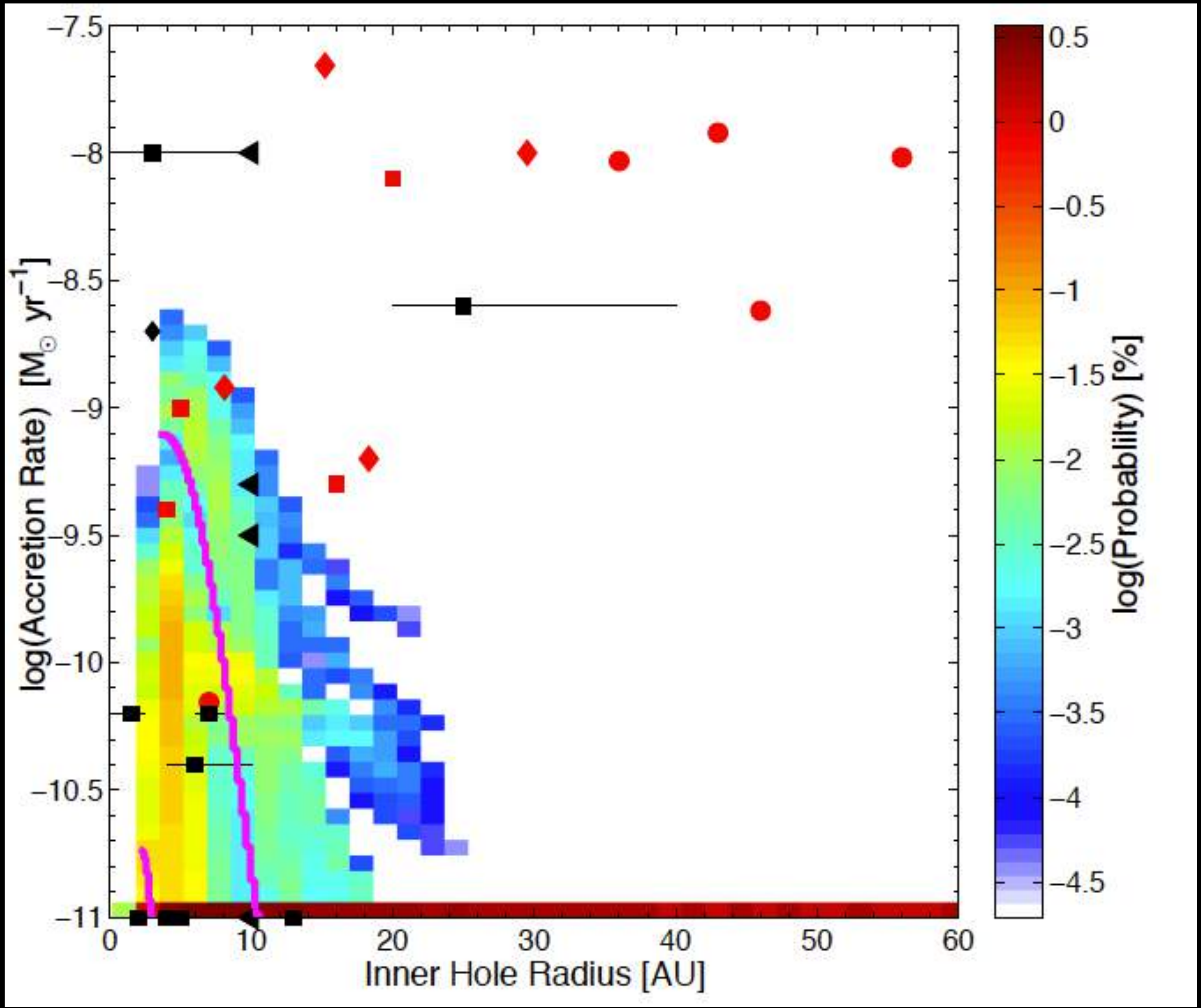


PHOTOEVAPORATION



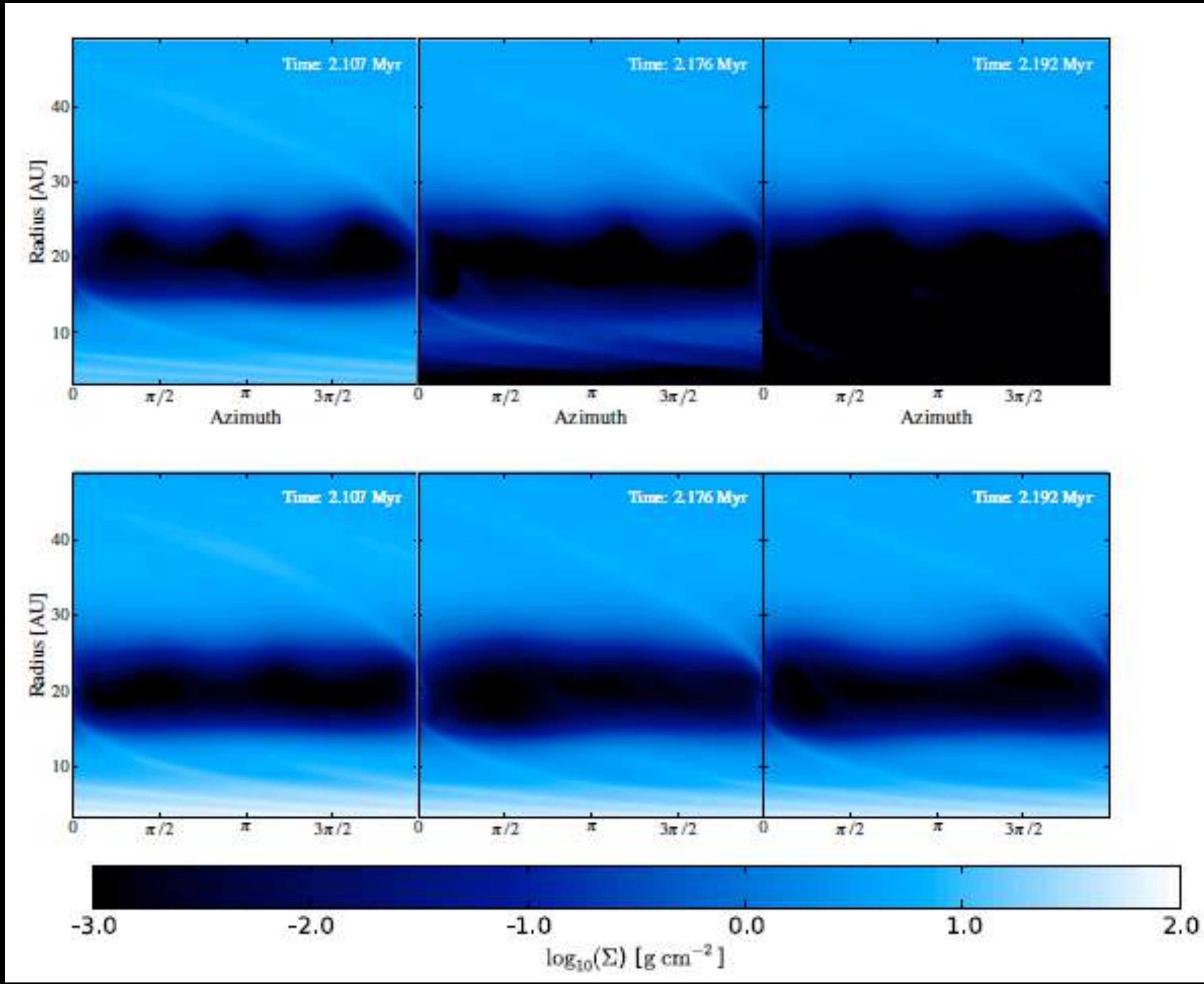
PHOTOEVAPORATION

- Struggles to produce transition discs with large gaps and high accretion rates. By the time that the disc gap is large enough the accretion rates in the inner disc have slowed or even quenched.
- Probably caused by giant planets, but grain growth can partially mimic this effect too.



PHOTOEVAPORATION

- Struggles to produce transition discs with large gaps and high accretion rates. By the time that the disc gap is large enough the accretion rates in the inner disc have slowed or even quenched.
- Probably caused by giant planets, but grain growth can partially mimic this effect too.
- Combination of the two: planet-induced photoevaporation

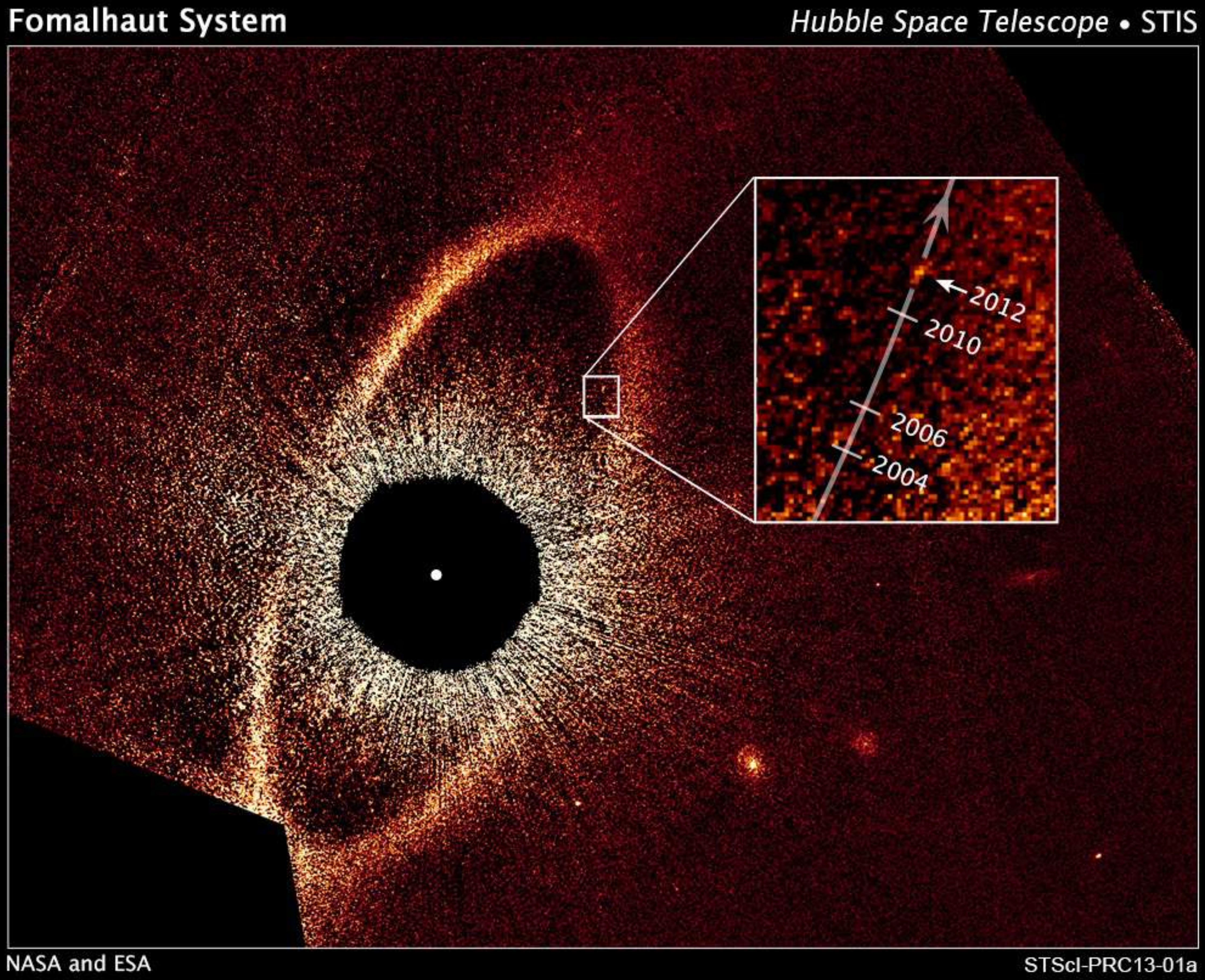


FROM UNIVERSE

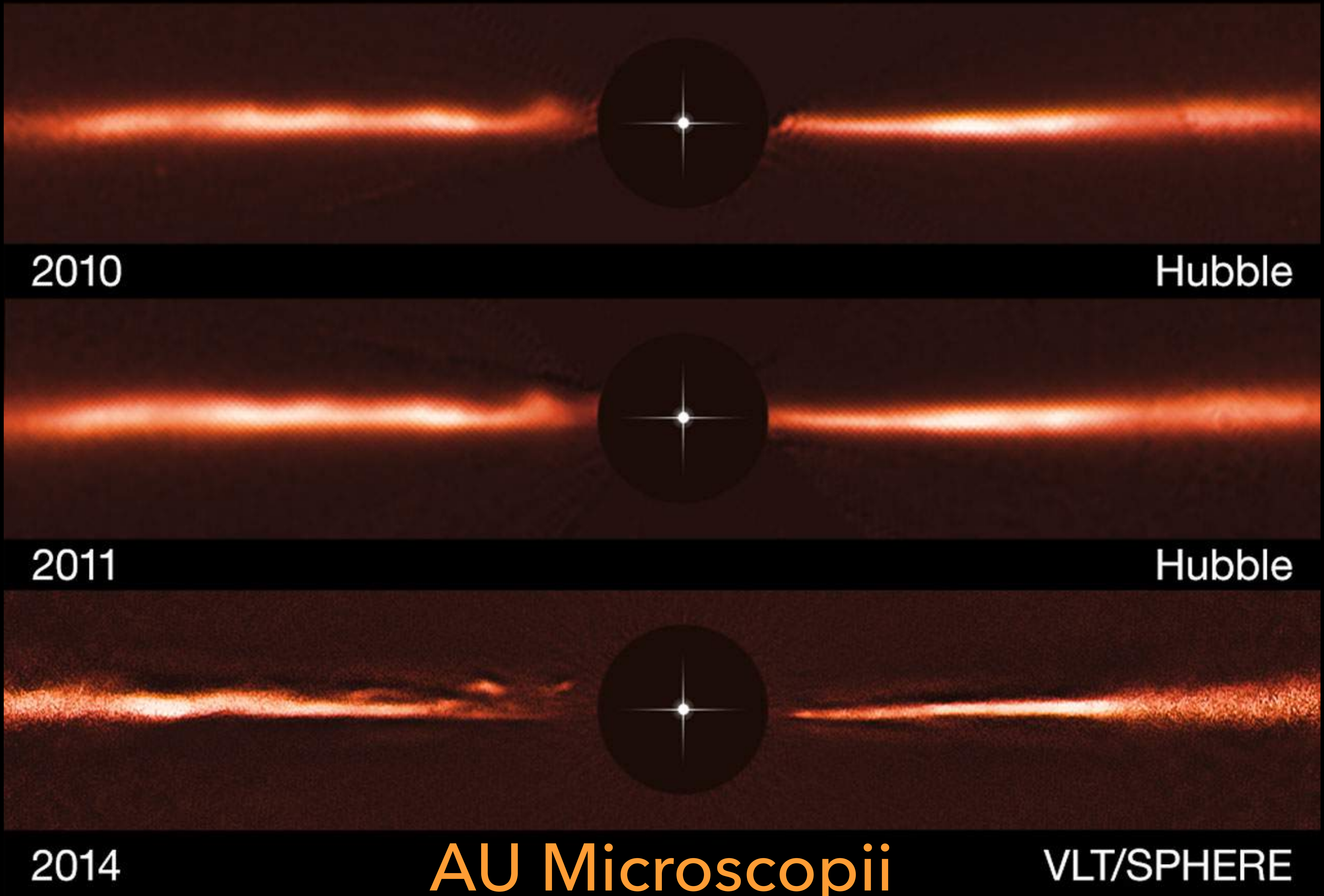
TO PLANETS

LECTURE 4.4: DEBRIS DISCS & COLLISIONS

DEBRIS DISCS

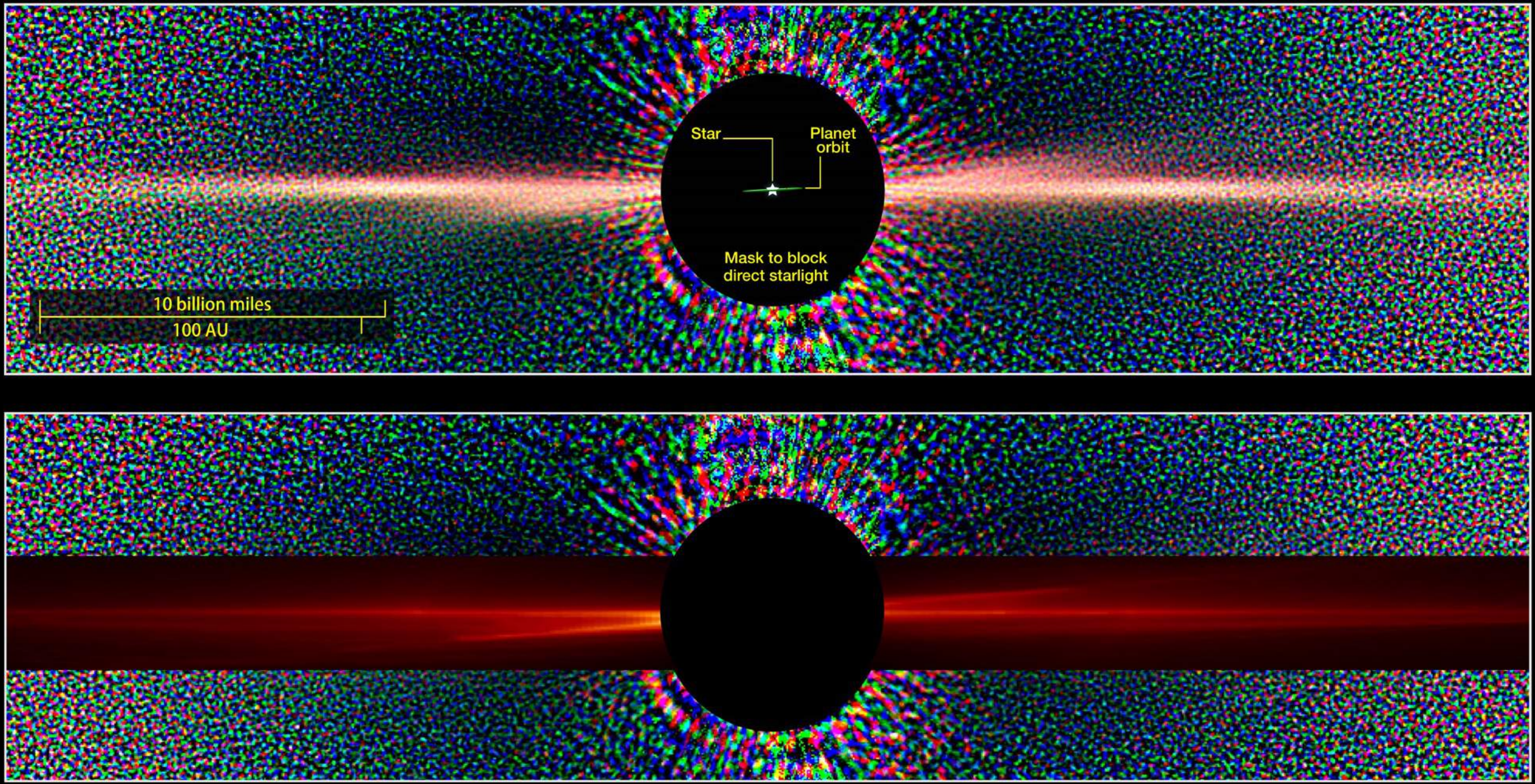


DEBRIS DISCS



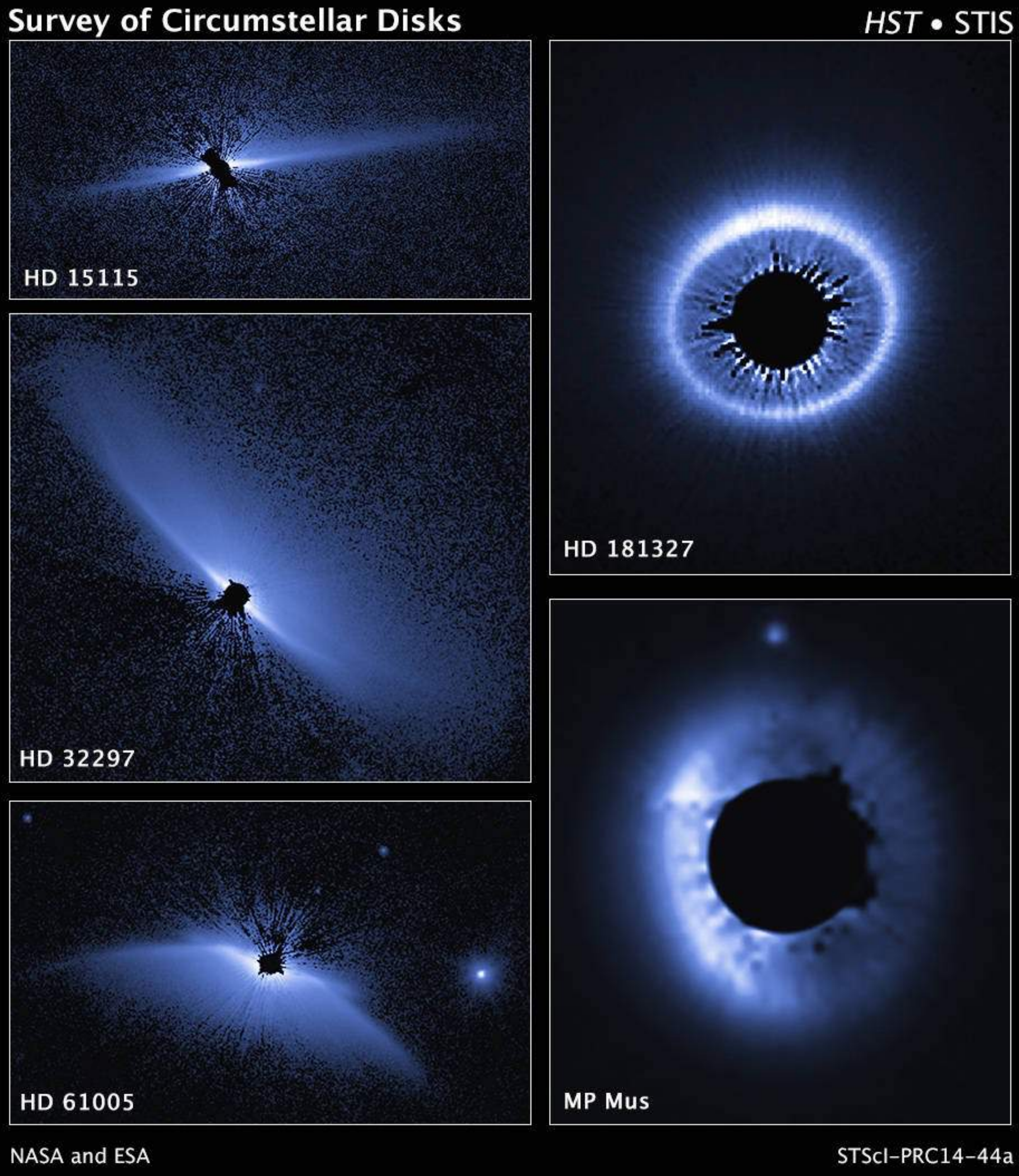
AU Microscopii

DEBRIS DISCS

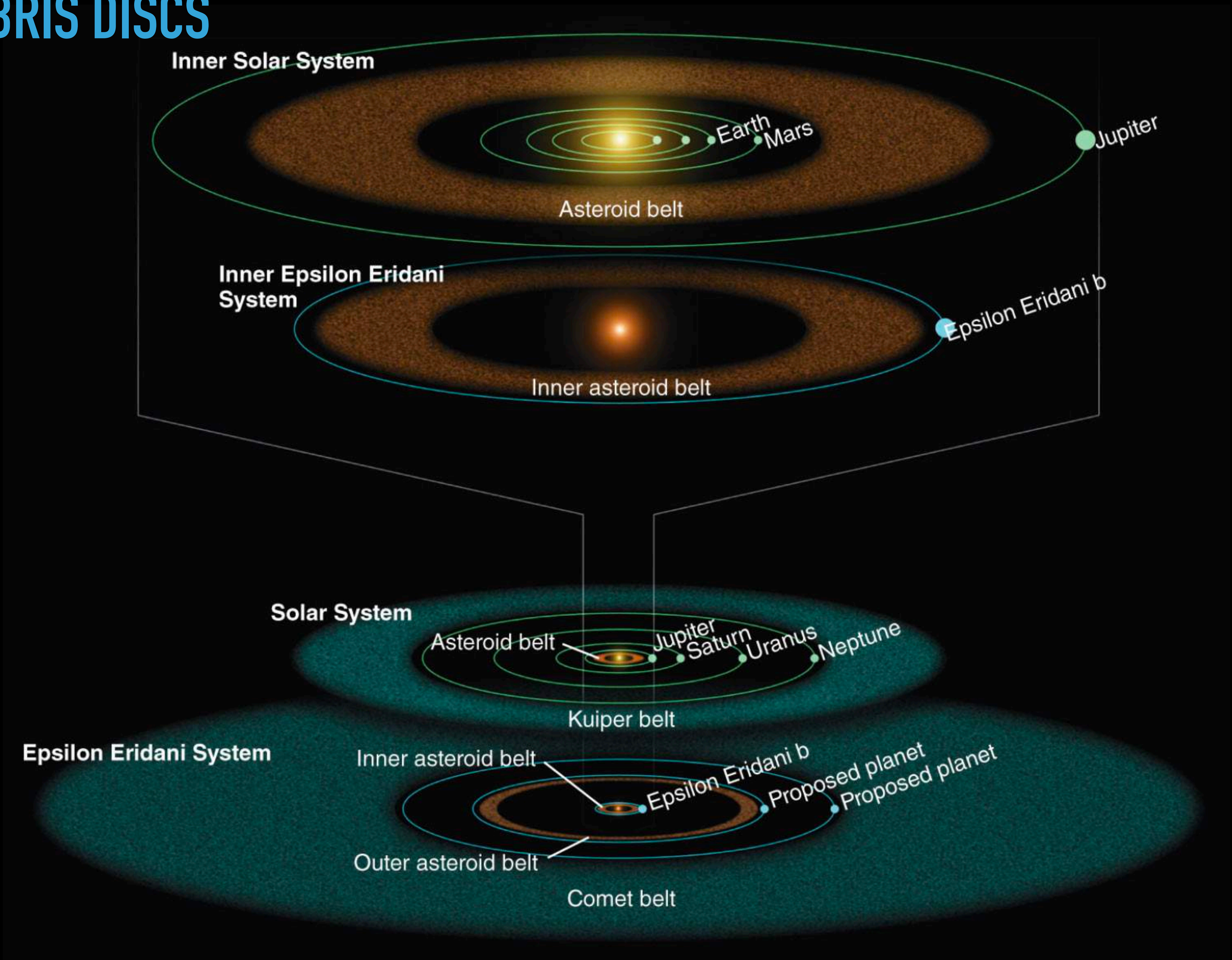


Beta Pictoris

DEBRIS DISCS



DEBRIS DISCS



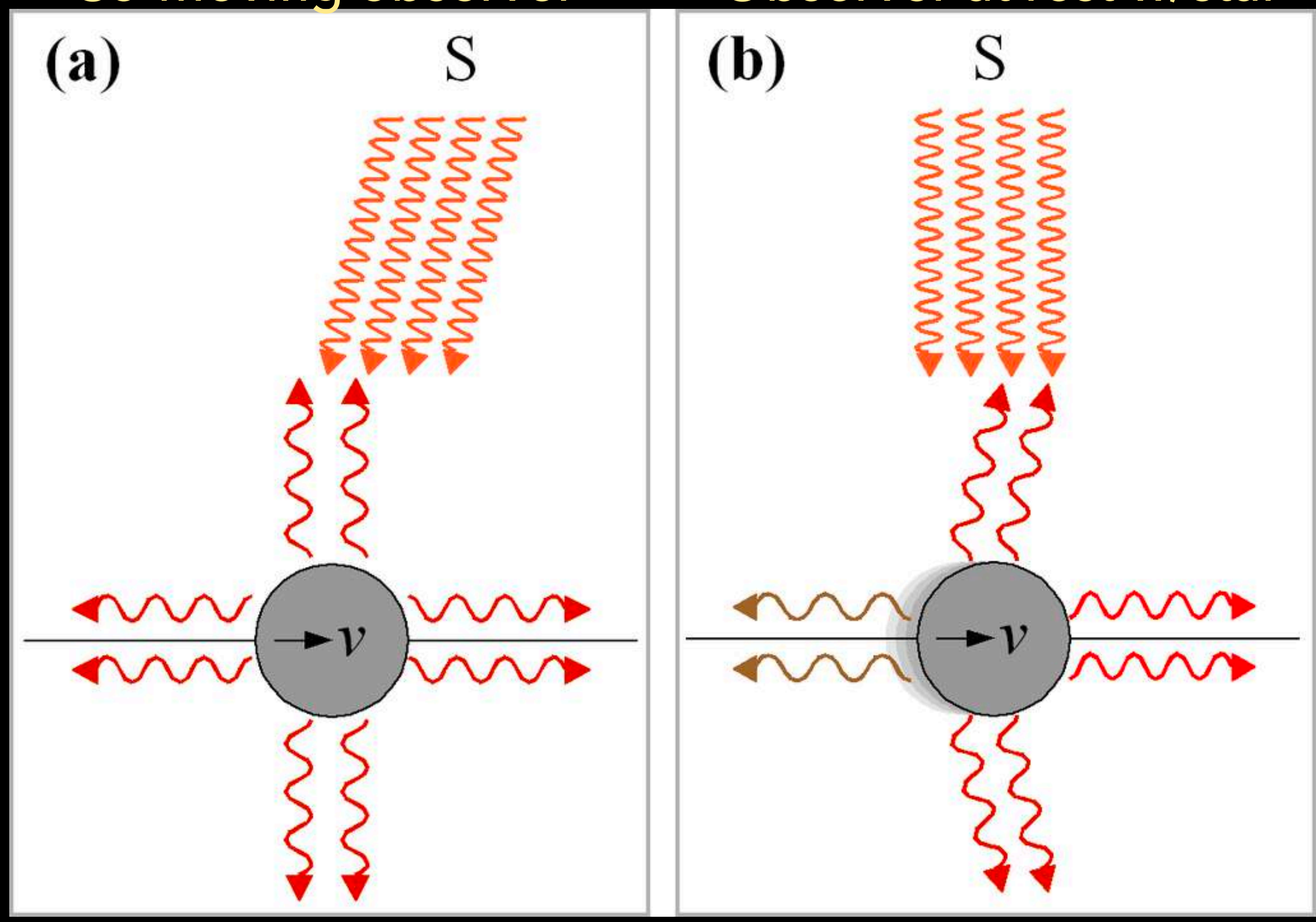
DEBRIS DISCS

- Debris discs are made from collisional grinding of leftover km-sized planetesimals.
- Collisions are destructive, producing a collisional cascade that repopulates small grain sizes (observable).
- $M_{\rm disc} \ll 0.01 M_*$
- $L_{\rm disc} \ll L_*$
- Dust and gas dynamics decoupled ($M_{\rm gas} < 10\ M_{\rm dust}$).
- Lifetimes depend on the stability of the system, size of the remnant disc, and the amount of stirring (Myr–Gyr).

DEBRIS DISCS

- Poynting–Robertson drag: stellar radiation causes a dust grain orbiting a star to lose angular momentum → radial drift. This is related to radiation pressure tangential to the grain's motion.
- (a) Stellar radiation comes from forward direction, but grain radiates isotropically.
- (b) Stellar radiation hits the grain laterally, but the grain appears to radiate more in the forward direction.

Co-moving observer   Observer at rest w/star



DEBRIS DISCS

- PR drag most pronounced for small objects close to the star:

$$F_{\rm PR} = \frac{v}{c^2}W = \frac{a^2 L_*}{4c^2}\sqrt{\frac{GM_*}{R^5}}$$

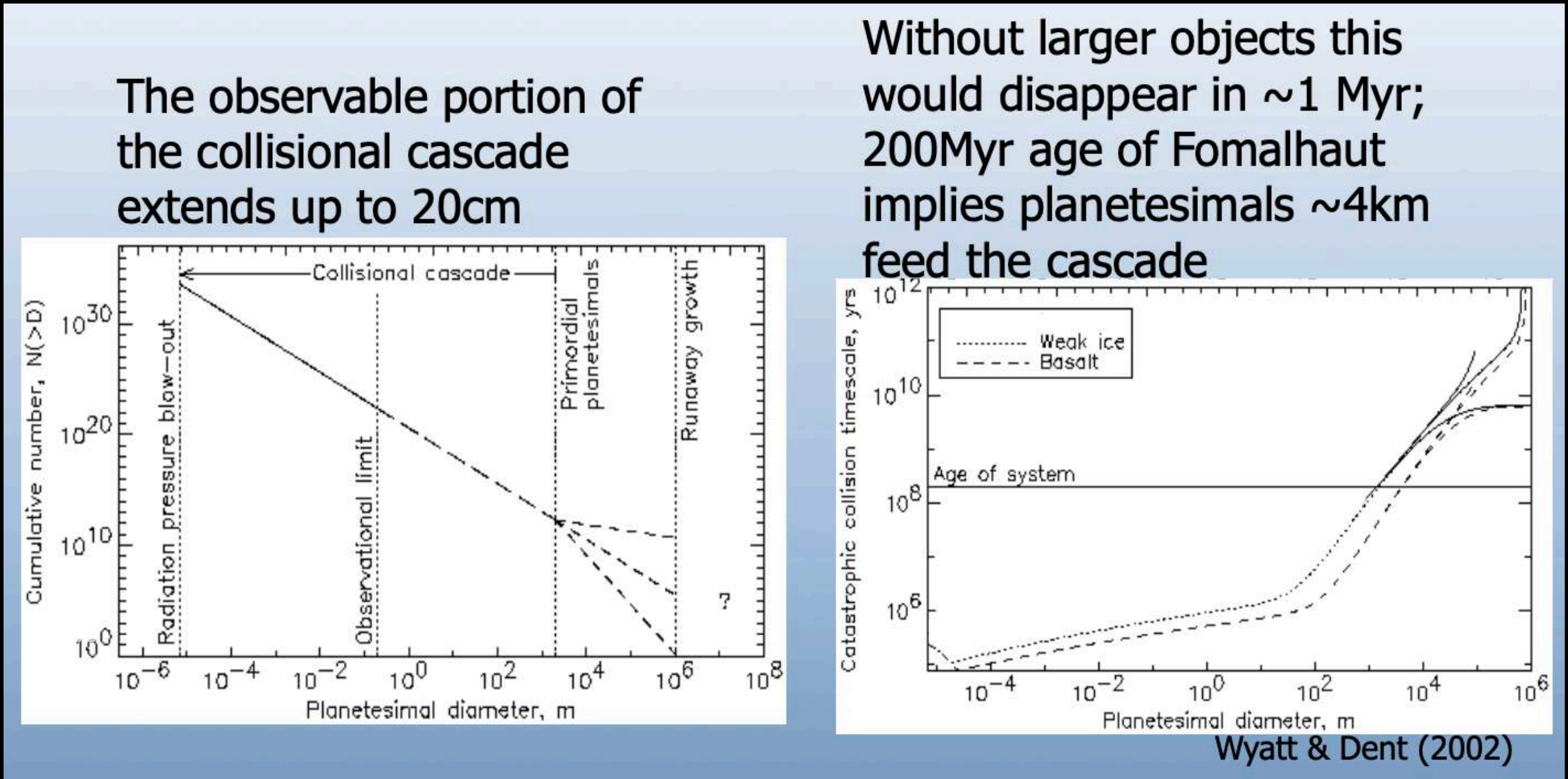
- Larger particles may not have constant surface temperature so radiation is not isotropic.
- Radiation pressure affects the effective force of gravity on the particle. Blows small particles away from the star and reduces the effective gravity of larger grains.

$$\beta = \frac{F_{\rm r}}{F_{\rm g}} = \frac{3L_* Q_{\rm PR}}{16\pi G M_* c \rho_{\rm grain} a}$$

- For $a \lesssim 1~\mu$m and $\beta \gtrsim 0.5$: particles are blown out
- $0.1 < \beta < 0.5$: spiral in/out depending on size (eccentric orbits).
- $\beta \approx 0.1$: spiral in (lifetime roughly $\propto \beta^{-1}$; 10000 yrs at 1 au).

DEBRIS DISCS

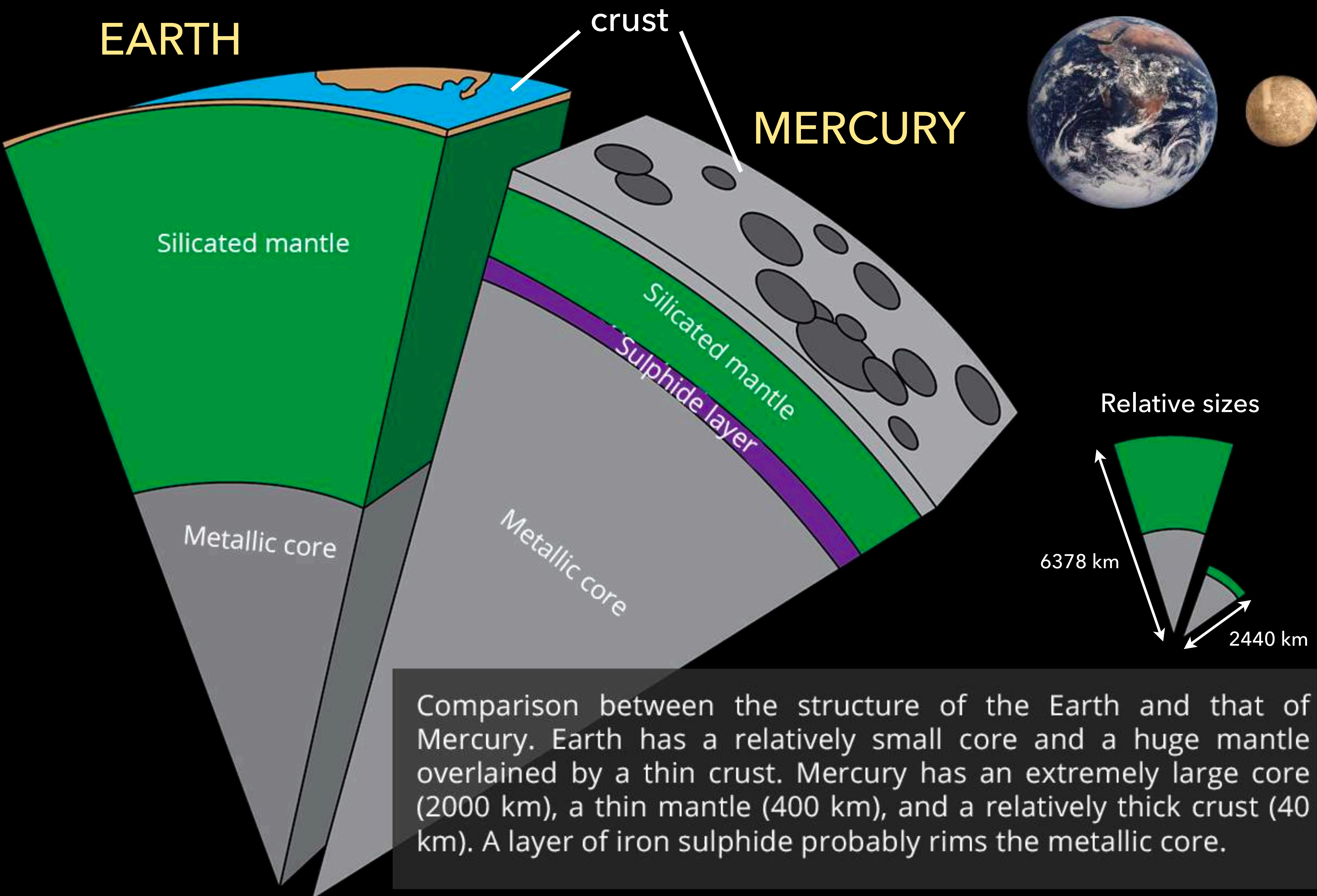
- Large grains and planetesimals contain most of the mass, but contribute little to no flux.
- Must characterise the invisible population of eroding parent bodies through modelling.



GIANT IMPACTS

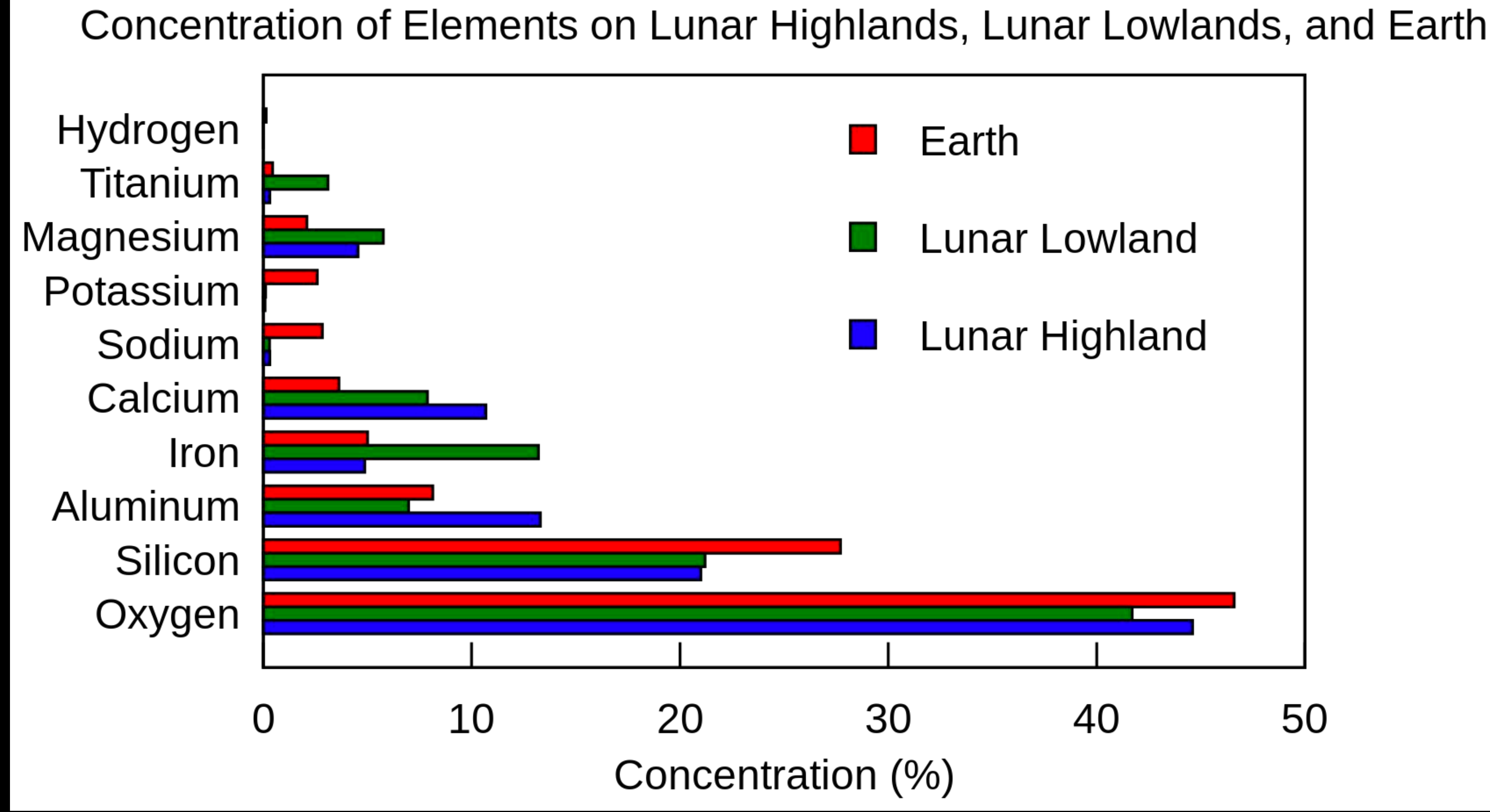
GIANT IMPACTS

- The late stages of the collisional accretion of planets involves collisions between planetary-sized bodies. These giant impacts involve enormous amounts of energy and are probably responsible for a number of particularities in the solar system:
    - Anomalous density of Mercury
    - Earth-Moon similarities
    - The topography of Mars
    - Tilt of Uranus' rotation axis
    - Existence of Charon (Pluto's moon).
- It is the last giant impact that leaves traces. The geological clock is reset in the impact region (molten surface).

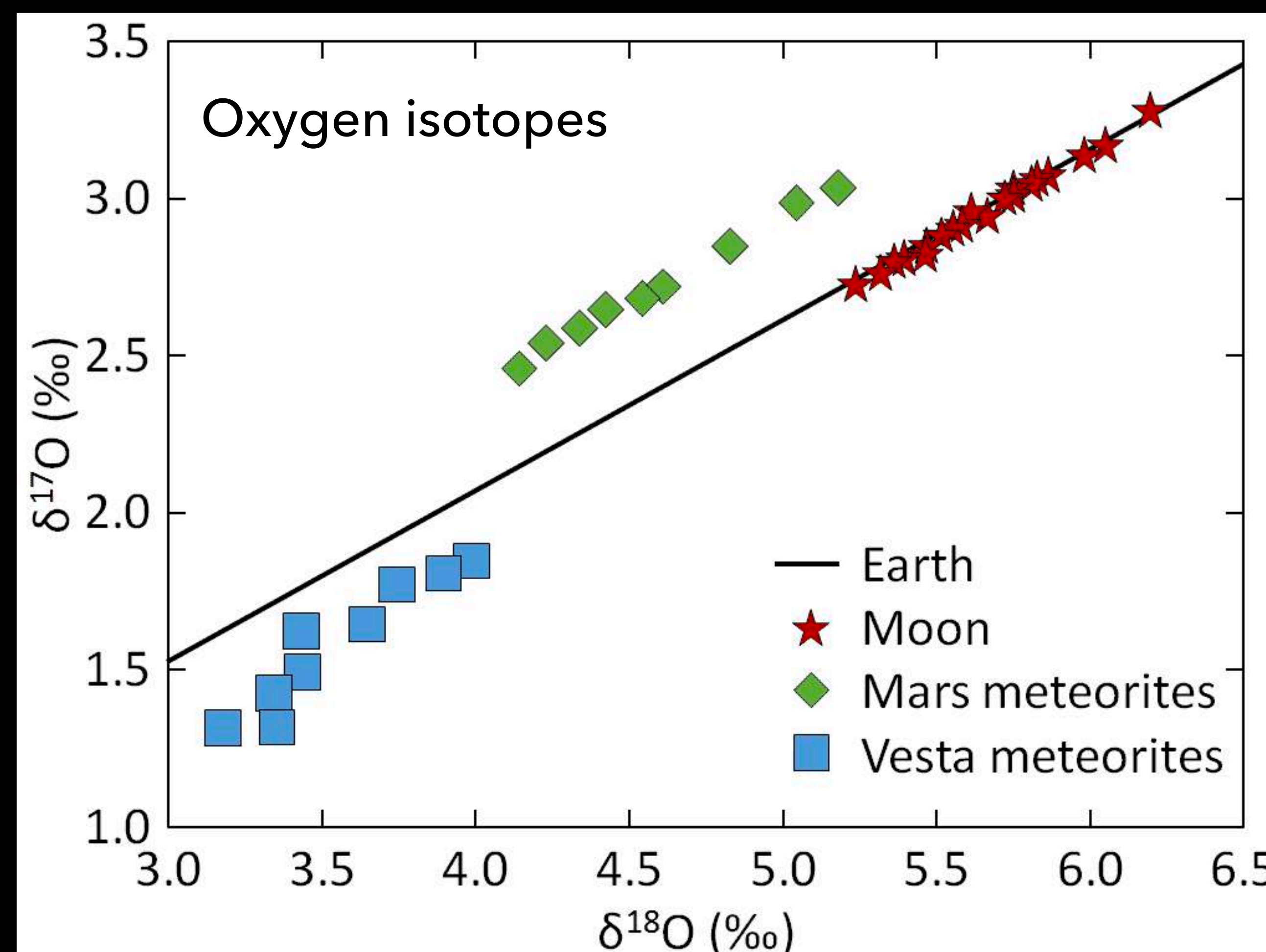


Comparison between the structure of the Earth and that of Mercury. Earth has a relatively small core and a huge mantle overlained by a thin crust. Mercury has an extremely large core (2000 km), a thin mantle (400 km), and a relatively thick crust (40 km). A layer of iron sulphide probably rims the metallic core.

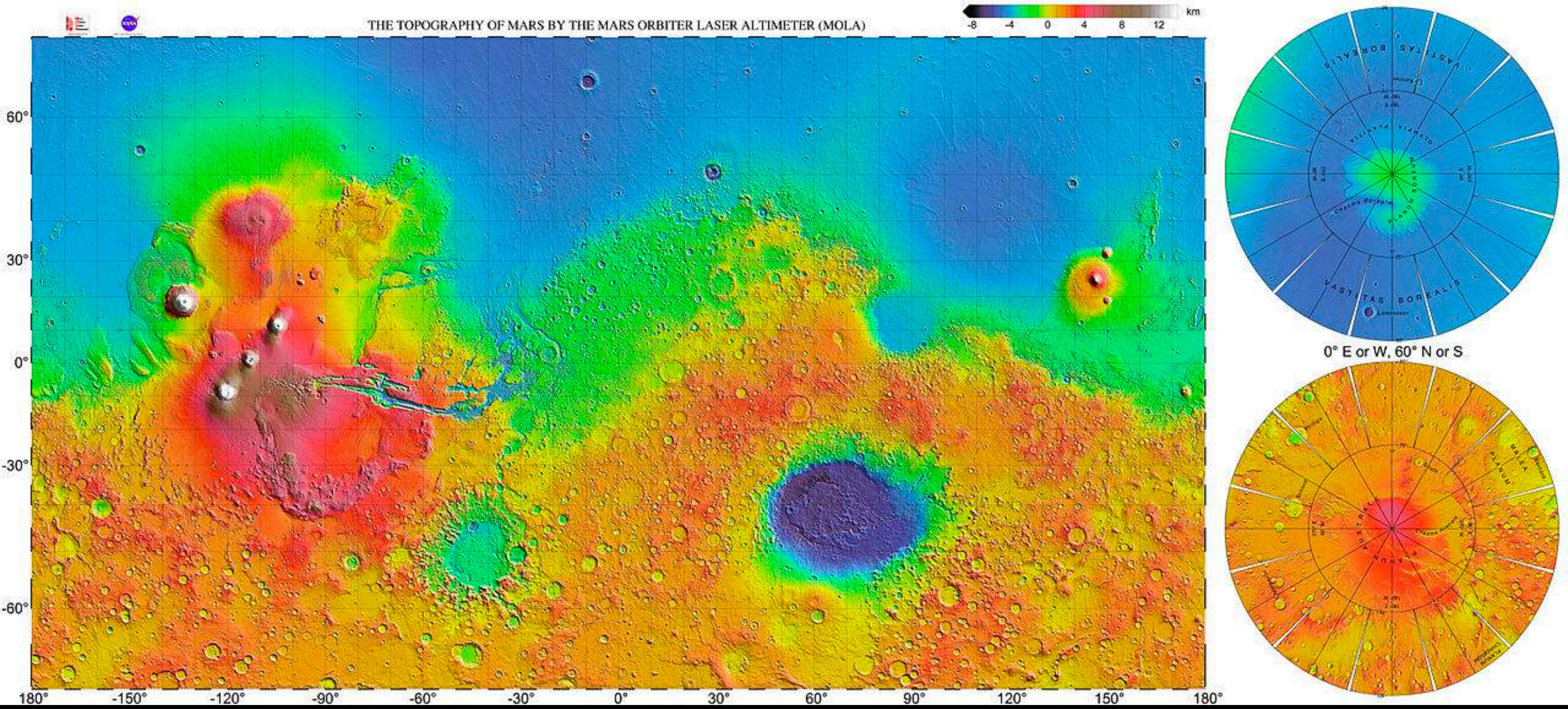

EARTH
Image Credit: NASA/JPL/USGS

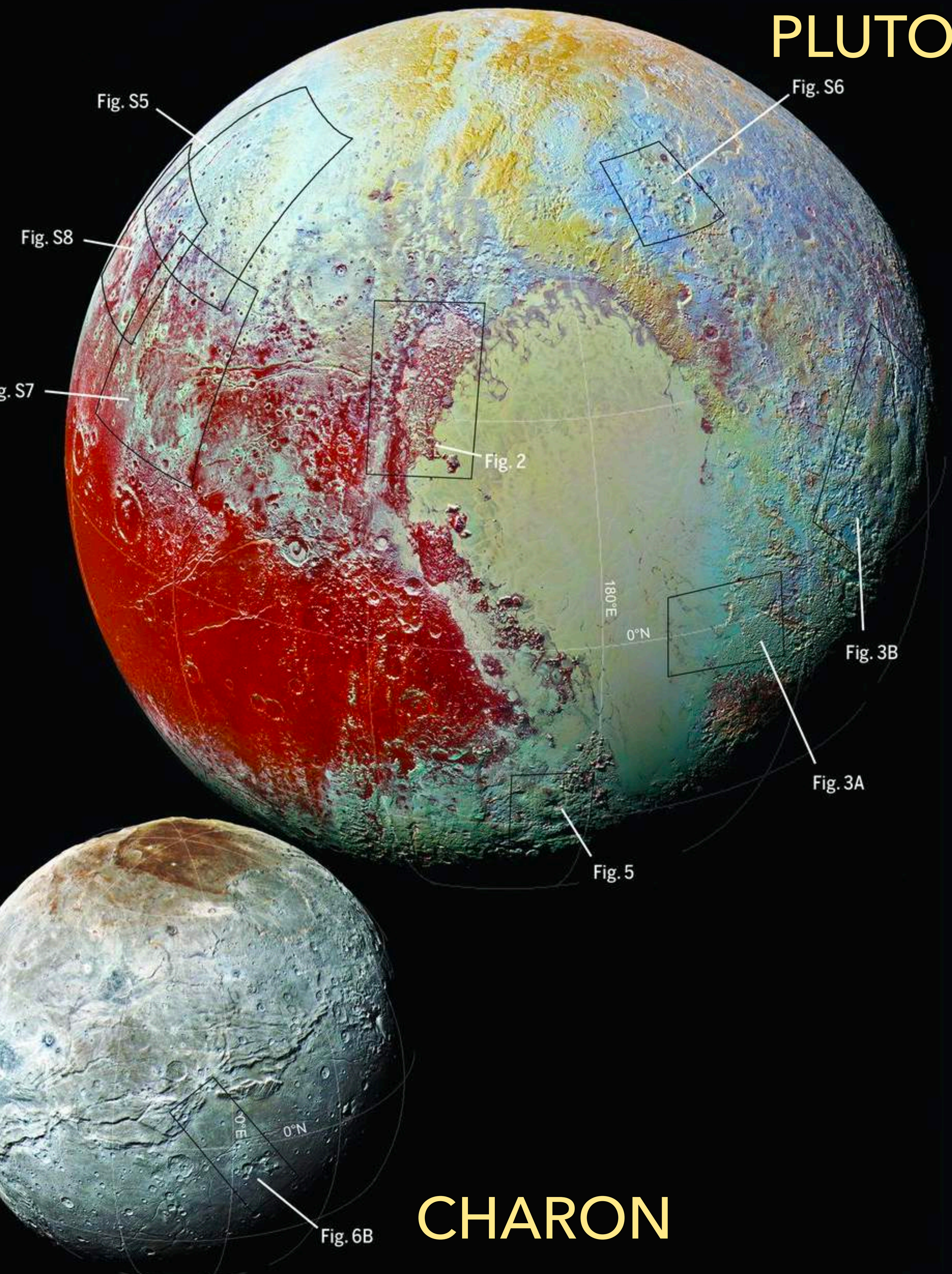

Concentration of Elements on Lunar Highlands, Lunar Lowlands, and Earth
Hydrogen
Titanium
Magnesium
Potassium
Sodium
Calcium
Iron
Aluminum
Silicon
Oxygen
Earth
Lunar Lowland
Lunar Highland
0
10
20
30
40
50
Concentration (%)

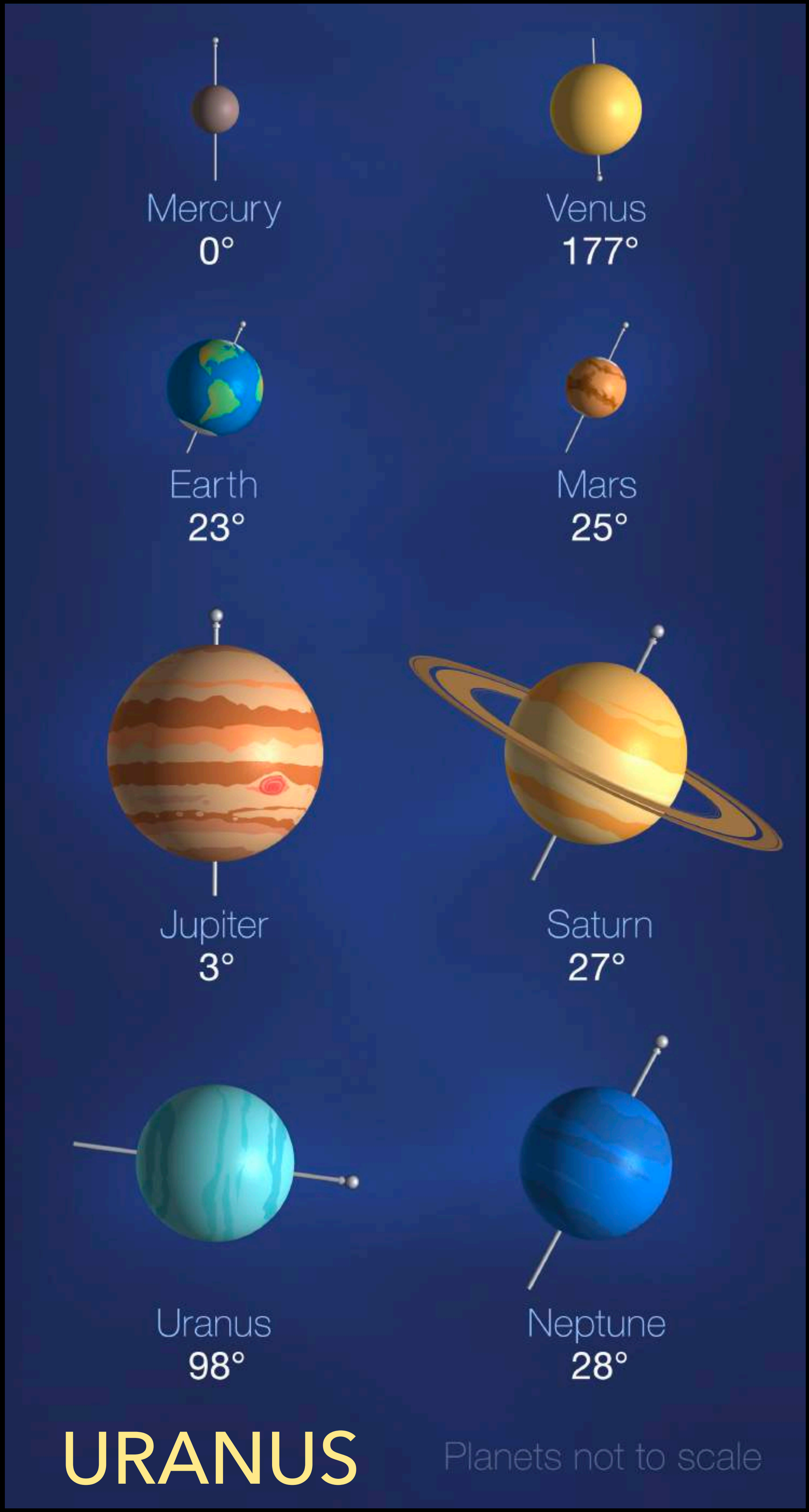

MOON


Oxygen isotopes
3.5
3.0
2.5
2.0
1.5
1.0
δ17O (‰)
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
δ18O (‰)
Earth
Moon
Mars meteorites
Vesta meteorites

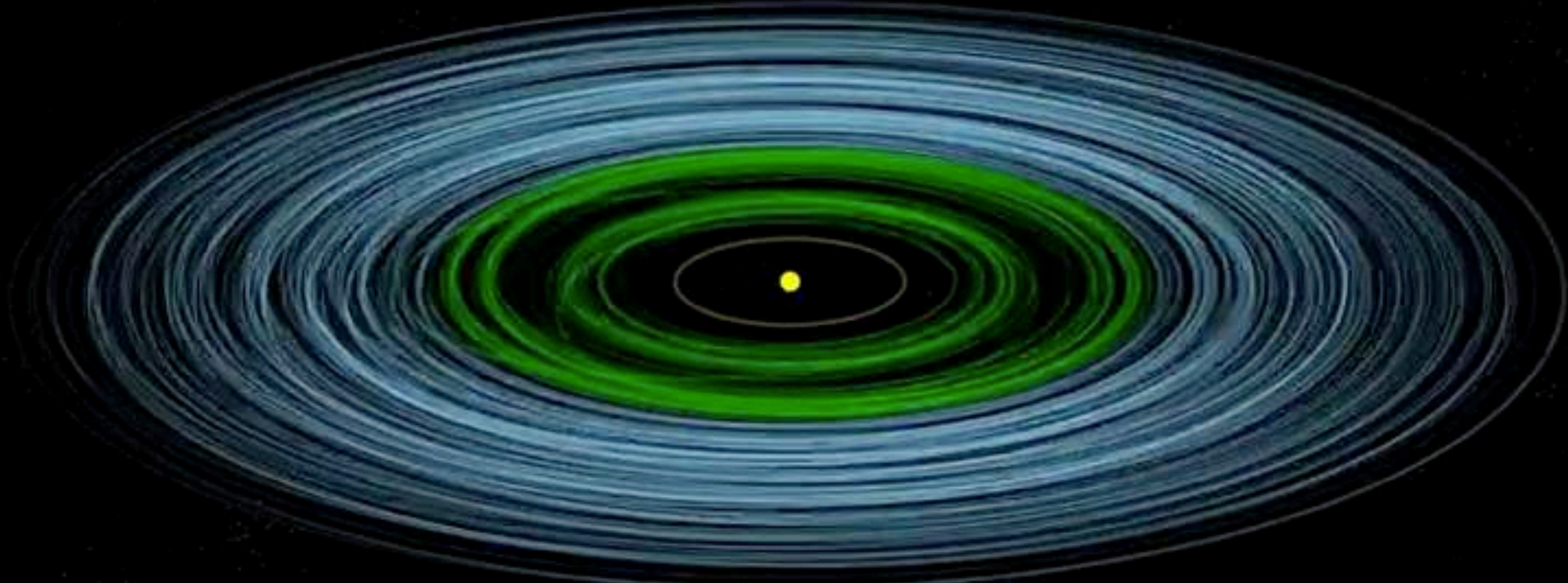
MARS


THE TOPOGRAPHY OF MARS BY THE MARS ORBITER LASER ALTIMETER (MOLA)
km
-8
-4
0
4
8
12
60°
30°
0°
-30°
-60°
180°
-150°
-120°
-90°
-60°
-30°
0°
30°
60°
90°
120°
150°
180°
VASTITAS BOREALIS
0° E or W, 60° N or S


PLUTO
Fig. S5
Fig. S6
Fig. S8
g. S7
Fig. 2
180°E
0°N
Fig. 3B
Fig. 3A
Fig. 5
0°E
0°N
Fig. 6B
CHARON


Mercury
0°
Venus
177°
Earth
23°
Mars
25°
Jupiter
3°
Saturn
27°
Uranus
98°
Neptune
28°
URANUS
Planets not to scale

LONG TERM EVOLUTION

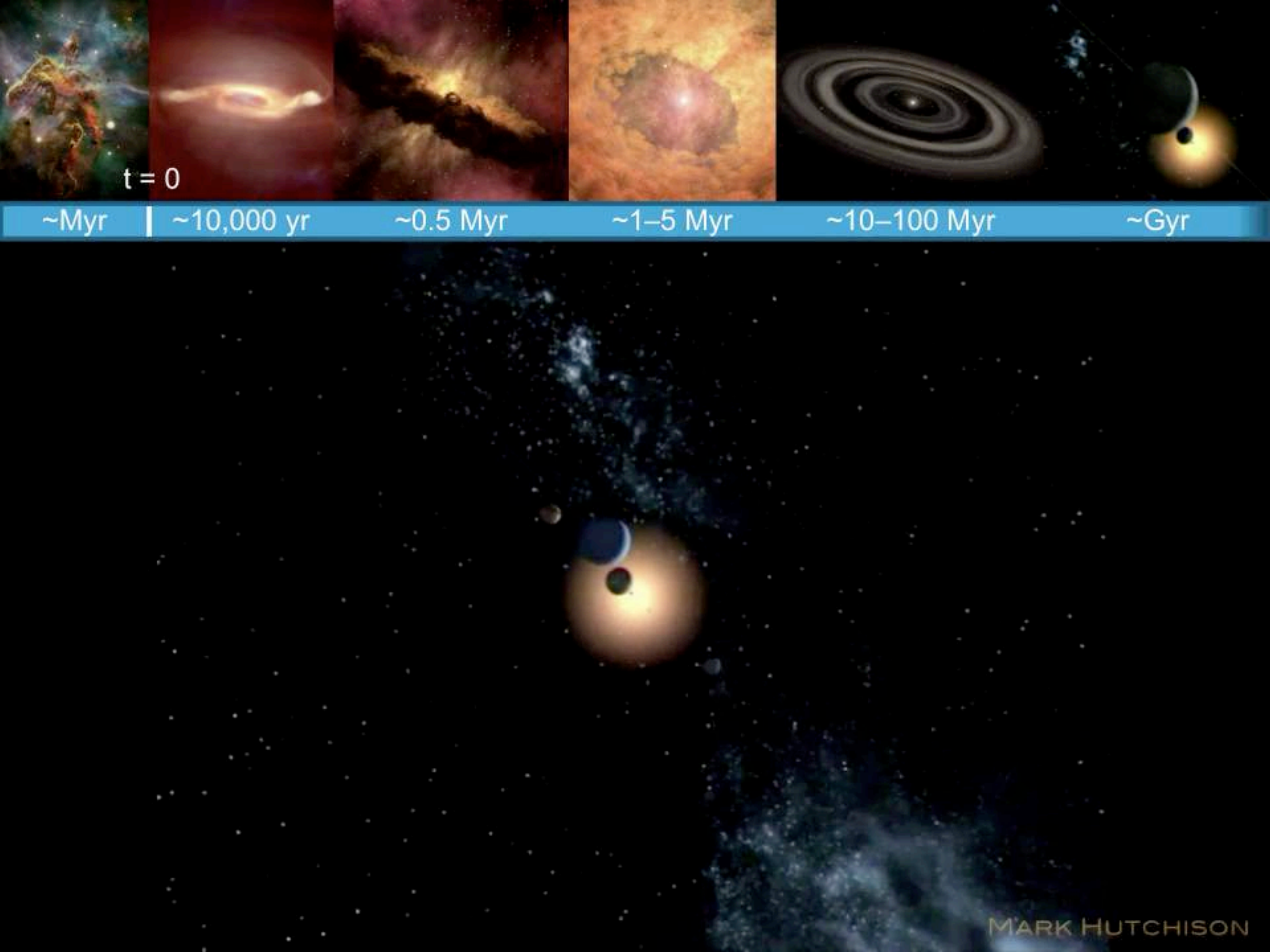


SUMMARY 1/2

- Random velocities are the result of viscous stirring, dynamical friction, damping. Together they determine the growth timescales of planetary cores.
- Largest cores experience runaway growth followed by slower oligarchic growth, limited by stirring and the isolation mass.
- Runaway gas accretion begins once the core reaches a critical mass of $10\text{–}20\ M_\oplus$.
- Embedded planets migrate rapidly through the disc (type I migration). Once the planet is large enough, it opens a gap and the migration slows (type II migration).
    - Result of torques exerted by spiral density waves.

SUMMARY 2/2

- Disc lifetimes are shortened by thermal winds (photoevaporation).
- Once the viscous accretion rate is comparable to the wind mass loss rate, a gap in the inner disc forms. The inner disc drains onto the star and the outer disc is quickly removed from the inside out.
- The remaining debris disc collisionally evolves for long time periods. Planets continue to sweep up and eject material.
- Giant impacts likely occur early on. Planets stir up debris from the outer disc and send them on eccentric orbits. Important water delivery mechanism.


t = 0
~Myr
~10,000 yr
~0.5 Myr
~1–5 Myr
~10–100 Myr
~Gyr
MARK HUTCHISON